Student ID: .....

TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

VICTORIA
UNIVERSITY OF WELLINGTON

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NOTE!!
THIS
CONTAINS
SOLUTIONS
*****

**EXAMINATIONS – 2013**

**TRIMESTER 2**

**COMP 103**
**INTRODUCTION TO**
**DATA STRUCTURES**
**AND ALGORITHMS**

**Time Allowed:** THREE HOURS

**Instructions:** Closed Book.

Attempt ALL Questions.

Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found.

The exam will be marked out of 180 marks.

Documentation on some relevant Java classes, interfaces, and exceptions can be found at the end of the paper. You may tear that page off if it helps.

There are spare pages for your working and your answers in this exam, but you may ask for additional paper if you need it.

Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination.

Non-electronic foreign language dictionaries are permitted.

| **Questions** | **Marks** |
|---|---|
| 1. General questions | [11] |
| 2. Using collections | [25] |
| 3. Implementing a Collection | [28] |
| 4. Recursion and Sorting | [16] |
| 5. Linked Structures | [22] |
| 6. Trees | [26] |
| 7. Binary Search Trees | [20] |
| 8. Priority Queues and Heaps | [20] |
| 9. Various other questions | [12] |

**SPARE PAGE FOR EXTRA ANSWERS**

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

## Question 1. General questions [11 marks]

**(a)** [3 marks] A Queue differs from a Set in three main ways. What are these differences?

Queue can have duplicates
Queue has an ordering
Queue is only accessible from one end

**(b)** [2 marks] What is a natural data structure to use, in order to reverse the order of items in a collection?

Stack.

**(c)** [2 marks] What is wrong with using the length attribute of a String as its hash code?

Will map two strings to the same hashcode, generating a collision.

**(d)** [2 marks] For a SortedArraySet, the complexity of contains() is $\mathcal{O}(\log n)$. Why?

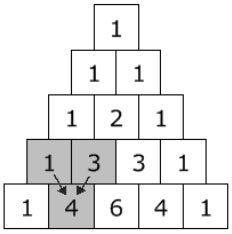
Being sorted, the Binary search algorithm can be used to FIND where the item is, or should be.

**(e)** [2 marks] For a SortedArraySet, is the complexity of the remove operation the same as for contains? Explain why or why not.

No it is not. Like contains, Removing starts with find, but we must then shift the items above the removed one down, to keep the array continuous and sorted. So it's O(n) instead.

## Question 2. Using collections [25 marks]

The figure below shows the first few rows of a pattern known as *Pascal's triangle*, which has many interesting mathematical properties. Each row is formed by summing 2 numbers from the row above, except at the edges, where there's a 1.




**(a)** [10 marks]

Suppose we represent one (horizontal) line of Pascal's triangle in Java using a *List*.

Complete the method generateNextList that takes one line (as a *List* of integers) as its argument, and returns the next line (also as a *List* of integers).
For example, if called with the list 1, 3, 3, 1, it should return the list 1, 4, 6, 4, 1.

```
public List <Integer> generateNextList(List<Integer> current) {
    // make a new list
    List <Integer> newlist = new ArrayList<Integer> ();
    newlist.add(1);
    for (int i=0; i<current.size()-1; i++)
        newlist.add(current.get(i) + current.get(i+1));
    newlist.add(1);
    return newlist;









}
```

**(b)** [10 marks] Complete the method below, that constructs a *Pascal's Triangle* with a certain number of lines. It should take the number of lines as an argument, and should return the *Pascal's Triangle* as a *List* of *List*s.

Your method should make use of the generateNextList() method described in **(a)**.

```
public List <List<Integer>> generateTriangleNums(int nlines){
    List<List<Integer>> lists = new ArrayList<List<Integer>> ();
    List <Integer> oneline = new ArrayList<Integer> ();
    oneline.add(1);

    for ( int t=0; t<=nlines; t++) {
        lists .add(oneline);
        oneline = generateNextList(oneline);
    }
    return lists ;




}
```

**(c)** [5 marks] Complete the method below, which prints a *Pascal's triangle*. For example, when called as displayTriangleNums(6) it should generate the output shown below.
You should make use of the generateTriangleNums() method described in **(b)**.

```
1
1     1
1     2     1
1     3     3     1
1     4     6     4     1
1     5    10    10     5     1
1     6    15    20    15     6     1
```

```
public void displayTriangleNums(int numlines){

    List <List<Integer>> theNums = generateTriangleNums(numlines);
    for ( List<Integer> alist : theNums) {
        for ( int j : alist )
            System.out.printf("%3d ", j);
        System.out.printf("\n");
    }




}
```

## Question 3. Implementing a Collection [28 marks]

The space agency *NASA* wants to send a probe into deep space, and the probe's onboard code uses Java arrays. Unfortunately cosmic rays occasionally corrupt the contents of these arrays, changing their values at random. Since the mission has very high cost, *NASA* opts to use an ArrayList implementation that is very robust: it will keep **three arrays** for a *List* instead of the usual **one**.

Here is partial code for their RobustArrayList implementation of the *List* interface.

The questions will ask you to complete the code for the methods isEmpty, add, get and ensureOneCapacity.

---

```
import java.util.*;

public class RobustArrayList<E> implements List<E> {
    private static int INITIALCAPACITY=2;
    private int count=0;
    private E[] data1, data2, data3;

    public RobustArrayList() {
        data1 = (E[]) new Object[INITIALCAPACITY];
        data2 = (E[]) new Object[INITIALCAPACITY];
        data3 = (E[]) new Object[INITIALCAPACITY];
    }


    :
    :
    :
    // Other public methods (isEmpty, size, set, get, add, remove, etc) would go here
    :
    :
    :



     private void ensureCapacity(){
         data1 = ensureOneCapacity(data1);          // see below
         data2 = ensureOneCapacity(data2);
         data3 = ensureOneCapacity(data3);
     }

     // The ''ensureOneCapacity'' method would go here also.
     :
}
```

**(a)** [3 marks] Complete the method isEmpty() in the box below:

```
public boolean isEmpty(){
    return (count == 0);

}
```

**(b)** [8 marks] Complete the method get() for RobustArrayList. If the same value occurs in any two of the three arrays, that value that should be returned. Otherwise the method should return null.

```
public E get(int index){

    if ((index < 0) || (index >= count))
        throw new NoSuchElementException();

    /* Lots of alternative ways for this. Here, we search for the first agreement we can find. */
    if (data1[index] == data2[index]) return data1[index];
    if (data1[index] == data3[index]) return data1[index];
    if (data2[index] == data3[index]) return data2[index];
    return null;









}
```

**(c)** [8 marks] Complete the method add() for RobustArrayList, which adds a new item at the specified position. Note that this is the "add at index" form of the operation, not "add at end".

```
public void add(int index, E item){
    if ((index < 0) || (index >= count))
        throw new NoSuchElementException();
    if (item == null)
        throw new IllegalArgumentException("adding null");

    ensureCapacity();
    for (int i=index+1; i<count; i++) data1[i] = data1[i-1];
    for (int i=index+1; i<count; i++) data2[i] = data2[i-1];
    for (int i=index+1; i<count; i++) data3[i] = data3[i-1];
    data1[index] = item;
    data2[index] = item;
    data3[index] = item;
    count++;
    return;









}
```

**(d)** [5 marks] Complete the method ensureOneCapacity(). If the array in the argument has room, a reference to it can be returned. Otherwise the method should return a reference to a new array that is larger but contains the original items.

```
private E[] ensureOneCapacity(E[] dat){
    int L = dat.length;
    if (count >= L) {
        E[] newdat =(E[]) new Object[2*L];
        for ( int j=0; j<L; j++) newdat[j] = dat[ j ];
        dat = newdat;
    }
    return dat;









}
```

**(e)** [4 marks] The above used three arrays to store each *List*. Suppose NASA wanted to make this even safer by allowing a mission-specific number of array copies to be maintained. The number of arrays could be passed as an argument to a new constructor, like this:

```
public RobustArrayList ( int NUMCOPIES) {
  :
}
```

Different space missions could then set NUMCOPIES to different values.

Outline the changes you would make to RobustArrayList to enable this to be done in a general way. Mention any problematic issues that would need to be addressed.

Instead of having data1, data2, etc, have a List of arrays.
A potential problem: what List implementation to use for this? Can't be a Robust one!

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Specify the question number for work that you do want marked.

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

## Question 4. Recursion and Sorting [16 marks]

**(a)** [4 marks] A method recHalf is supposed to print larger and larger chunks of a *String* passed as an argument. For example, recHalf("ABCDEFGH") should print the following:

```
A
AB
ABCD
ABCDEFGH
```

In the box below, give a *recursive* solution for this method.

*Hint:* Java String objects have a method substring(int beginIndex, int endIndex) that returns a new string that is a subsequence of the original. They also have a method length().

```
public void recHalf(String str) {

    if (str.length() > 1)
        recHalf(str.substring(0, str.length()/2));
    System.out.println(str);









}
```

**(b)** [2 marks] QuickSort is a much faster algorithm than InsertionSort in most circumstances. However, there is one case in which InsertionSort will work faster: what is that case?

If the list is already almost sorted, InsertionSort will completely sort it in only about n comparisons.

**(c)** [5 marks]

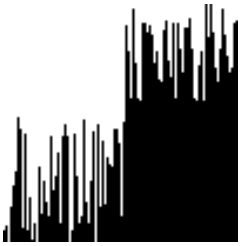
The figure on the right shows SelectionSort part of the way through the sorting operation. The figure shows a snapshot of the whole array being sorted, with the height of the dark bars representing the values of the respective array cells.
Suppose there are 32 items in the whole array.

Indicating your argument by words or a diagram, estimate how many more item-to-item comparisons SelectionSort will need to carry out from this point onwards, for it to finish the sorting operation completely. Give an actual number of steps, not the "big-O" complexity.

Has about 16+15+14+...+2+1 to go. This is half of 16*16, ie.16*8=128 comparisons to go

**(d)** [5 marks]

The figure on the right shows QuickSort part of the way through the sorting operation. Again, suppose there are 32 items in the array being sorted.
Indicating your argument by words or a diagram, estimate how many more item-to-item comparisons QuickSort will need to carry out from this point onwards, for it to finish the sorting operation completely.
For simplicity, you may assume that the pivot is almost always about half way through the relevant segment being sorted.



*Note:* The snapshot shown here was not taken at the same stage as the one for SelectionSort, so the number of steps to go is not necessarily going to be lower than your answer for (c).

Splitting according to the pivot takes n operations for a segment that is n items long. QuickSort has to do 2*16=32 of these. Each of THOSE has 2*8, ie 32 more. Each of THOSE has 2*4, ie 32 more. Each of THOSE has 2*2, ie 32 more. Each of THOSE has 2*1, ie 32 more. So QuickSort has about 5*32 = 160 comparisons still to go.

## Question 5. Linked Structures [22 marks]

Consider the following IntegerNode class:

```
public class IntegerNode {

   // The integer value stored by the node.
   private int value;

   // A reference to the next IntegerNode in the linked list.
   private IntegerNode next;
   ...
}
```

As usual, an IntegerNode can be regarded as the head of a linked list that is terminated by a null value in the next field of the last node in the list.

Your colleague Sam shows you the code he has drafted for a method to be added to the IntegerNode class that is supposed to add up the numbers in a linked list of IntegerNodes, at odd positions (i.e., 1, 3, 5, ...), with the first node being at position 1.

```
public int skippy() {
   int result = this.value;

   while (this != null) {
      result += this.value;
      this = this.next.next;
   }

   return result;
}
```

**(a)** [6 marks] There are several problems with his code. Very briefly point out all problems you can identify.

The value of the first node is used twice.
Cannot assign to this.
Will not work for lists with an odd number of elements.

**(b)** [5 marks] Write a method public IntegerNode lastButOne() to be added to IntegerNode that returns the reference to the node that precedes the last node in the linked list. You can assume that the list will always contain at least two elements.

```
public IntegerNode lastButOne() {




}
```

public IntegerNode lastButOne() if (this.next.next == null) return this;
return this.next.lastButOne();

**(c)** [5 marks] Write a method int calculate(int x) to be added to IntegerNode that treats the numbers in a linked list of IntegerNode as coefficients in a polynomial and returns the value of $n_1 + n_2 * x + n_3 * x^2 + n_4 * x^3 + \ldots$, with $n_1$ being the value for the first node, etc.

```
public int calculate( int x) {




}
```

public int calculate(int factor) if (next == null) return this.value;
return this.value + factor * this.next.calculate(factor);

**(d)** [6 marks] You asked your colleague Sam to write a LinkedIntegerList class that has an iterator() method and implements the Iterable interface. Sam comes back with a LinkedIntegerList design that supports a cursor that can be moved forwards and backwards between nodes and argues that this supports an even better form of iteration. Briefly discuss whether Sam has a point. Do you still want Sam to implement your initial design? Provide arguments to justify your answer.

Sam is right that a cursor supports more flexible navigation.
You still want Sam to implement the original design because you can then use the forEach loop style on LinkedIntegerList and multiple clients can iterate on a LinkedInteger List at the same time.

**SPARE PAGE FOR EXTRA ANSWERS**

Cross out rough working that you do not want marked.
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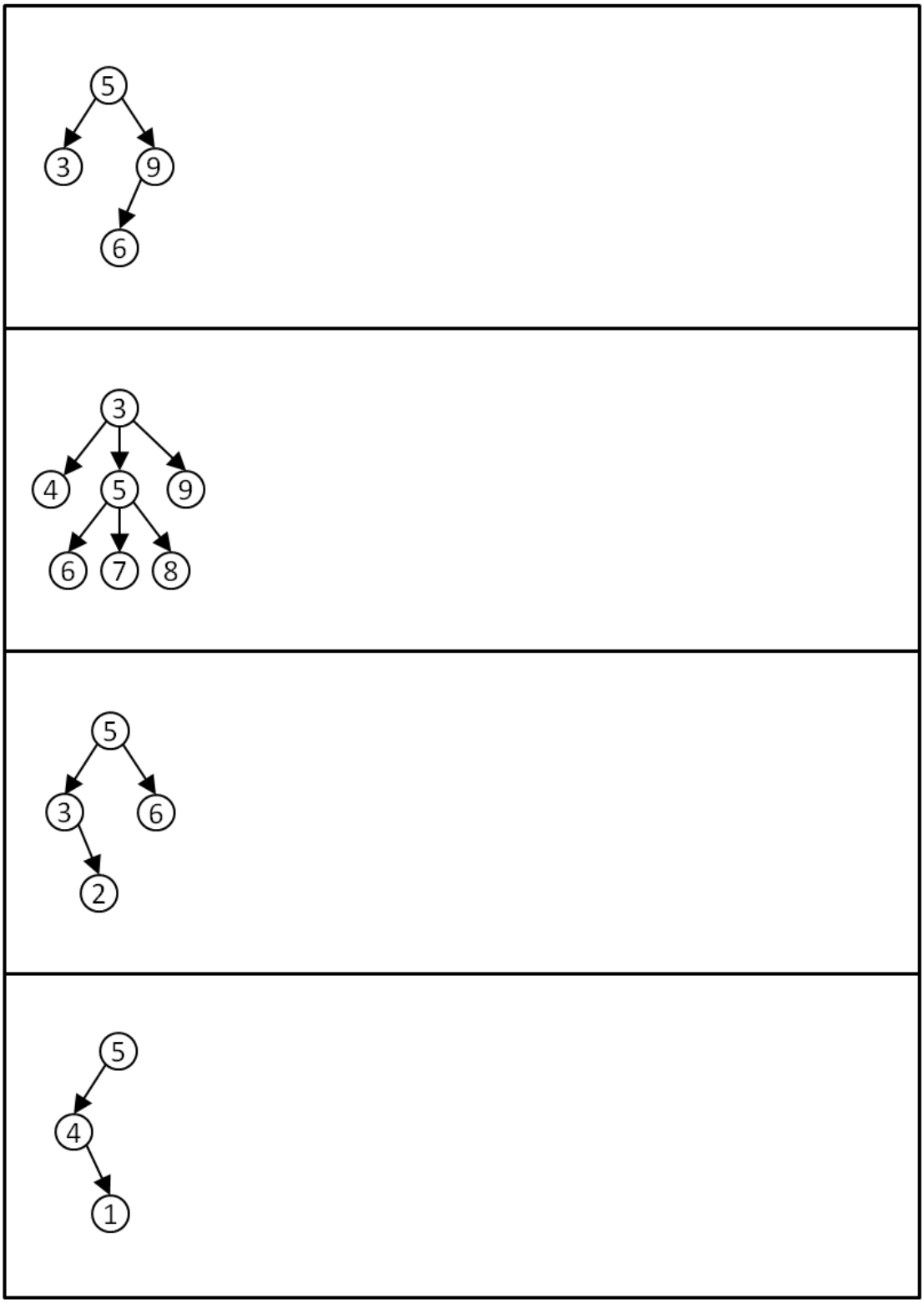
**Question 6. Trees** [26 marks]

**(a)** [8 marks] For each of the following four structures state which term – binary tree, binary search tree, partially ordered binary tree, general tree, or not a tree – best describes it and briefly justify your answer.



**(b)** [4 marks] Your colleague Sam has written the following method contains for class GeneralTreeNode to check whether the subtree formed by the receiver contains the argument node:

```
public class GeneralTreeNode {
  private String name;

  private LinkedList<GeneralTreeNode> children = new LinkedList<GeneralTreeNode>();

  ...

  public boolean contains(GeneralTreeNode node) {
      if (this == node)
        return true;

      for(GeneralTreeNode child : children) {
          child.contains(node);
      }

  }
```

Briefly point out any bugs you identify and suggest the correction required.

needs to return false at the end needs to check result of recursive ”contains” and return if it is true.

**(c)** [3 marks] Briefly explain why it is easier to implement a method that removes a node for class GeneralTreeNode compared to class BinarySearchTreeNode.

parent node can just accept all the children of the removed node A BST potentially requires restructuring.

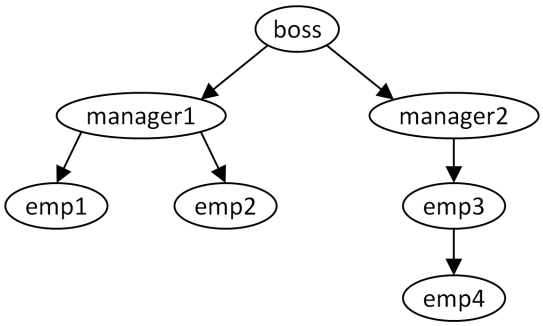
**(d)** [5 marks] Your colleague Jimmy has a terrible coding style. Try to understand what the following method for class GeneralTreeNode does, briefly describe its purpose, and state how it normally needs be invoked.

```
public void print( int td, int cd) {
  if (cd == td) {
    System.out.println(name);
    return;
  }

  for (GeneralTreeNode child : children)
    child . print (td, cd+1);
}
```

Prints all the nodes in one level.
root.print(targetLevel, 0);

Consider the following general tree composed of GeneralTreeNode nodes:




**(e)** [6 marks] Give an implementation of method toString() for class GeneralTreeNode so that it returns the string

*boss ( manager1 ( emp1 emp2 ) manager2 ( emp3 ( emp4 ) ) )*

when invoked on the root node of the above tree.

Class GeneralTree node is shown as part of Sam's code just before part **(b)**.

```
public String toString () {




}
```

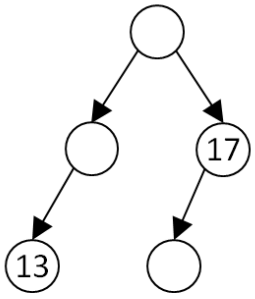
public String toString() String result = name;

if (!children.isEmpty()) result += " ( "; for (GeneralTreeNode child : children) result += child + " "; result += ")";

return result;

**Question 7. Binary Search Trees** [20 marks]

Consider the following (*incomplete*) binary search tree that stores integer values:




**(a)** [1 mark] Assuming the top node is referenced by root, what is the value of root.left().left().getValue()?

13

**(b)** [3 marks] Fill in the missing integer numbers into the blank nodes above.

**(c)** [2 marks] Write down the sequence of vales produced by a post-order traversal, assuming values have been filled in as necessary.

13, 14, 16, 17, 15

**(d)** [2 marks] Briefly describe a method of returning the largest value stored in a binary search tree without ever comparing two values with each other.

Start at the root and move down to the right as long as possible

**(e)** [2 marks] Your colleague Ben has implemented a remove method for a binary search tree implementation. For some reason he did not follow the COMP 103 notes and hence does not replace a node that has two children with the leftmost child of its right subtree. Instead he replaces it with the rightmost child of its left subtree.

Briefly state your response to Ben's design.

Ben, you are alright, bro. Whether you are using the in-order predecessor or the in-order success does not matter.

**(f)** [10 marks] Consider the following implementation for a binary tree containing integers:

```
public class BinaryIntegerTree
{
  private int value;

  private BinaryIntegerTree left ;
  private BinaryIntegerTree right ;
   ...
}
```

In the box below, complete a method "isBinarySearchTree" for this class. The method tests whether a tree satisfies the conditions for being a binary search tree.

You may assume that the range of values contained only spans $1 - 99$.

```
public boolean isBinarySearchTree(int min, int max)
 {
   if (value<=min || value >=max)
     return false;

   if ( left != null)
     if (! left .isBinarySearchTree(min, value))
       return false;

   if ( right != null)
     if (! right .isBinarySearchTree(value, max))
       return false;

   return true;
```

**Question 8. Priority Queues and Heaps** [20 marks]

**(a)** [2 marks] Your colleague Pete has started to write an implementation of a generic PriorityQueue class. He has required the element type to implement interface "*Comparable*".

Briefly discuss whether his requirement is reasonable.

Yeah, Pete knows his stuff!
Elements need to be comparable, otherwise the PQ has no idea what element come before another.

**(b)** [2 marks] Pete now suggests to extend class PriorityQueue with a constructor that accepts a *comparator* object.

Do you think Pete's suggestion is a good one? Justify your answer.

Pete rocks again.
Allows PQ to be used for multiple purposes.

**(c)** [3 marks] Pete is excited about using a heap for his priority queue implementation. He thinks he can use one of the four standard traversal strategies to traverse the underlying partially ordered tree so that the elements are yielded in the order of descending priority.

Do you think Pete's plan is going to work? Justify your answer.

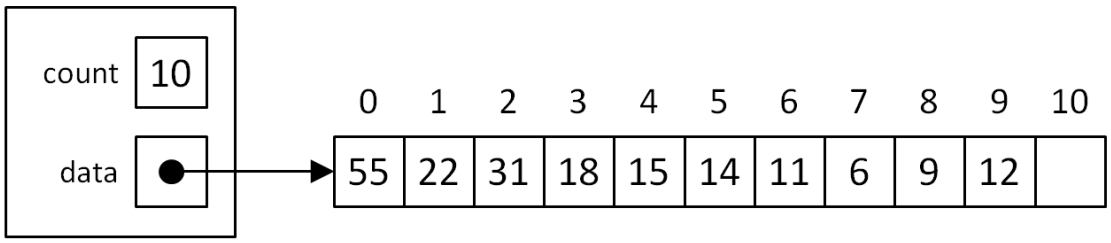
Hey Pete, think again.
Partially ordered trees are only partially ordered. There is no traversal strategy that yields all elements in order.

**(d)** [3 marks] Pete wants to add a constructor to class PriorityQueue that takes a collection of elements as an argument and populates the queue with them. Pete thinks he cannot beat $O(n \log n)$ as the asymptotic complexity for this constructor.

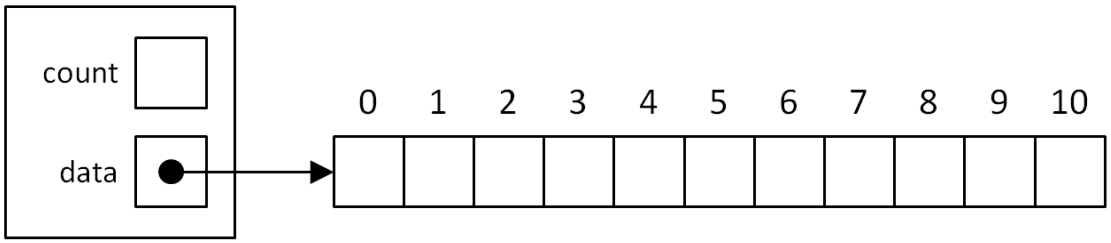
Do you think Pete is right? Justify your response.

For Pete's sake, have you forgotten about "heapify" and its $O(n)$ complexity?

Pete is not sure whether his implementation is correct and asks you for help. He shows you the following state of his priority queue which uses an internal array to store items in a heap. In his implementation elements with larger values have higher priority.

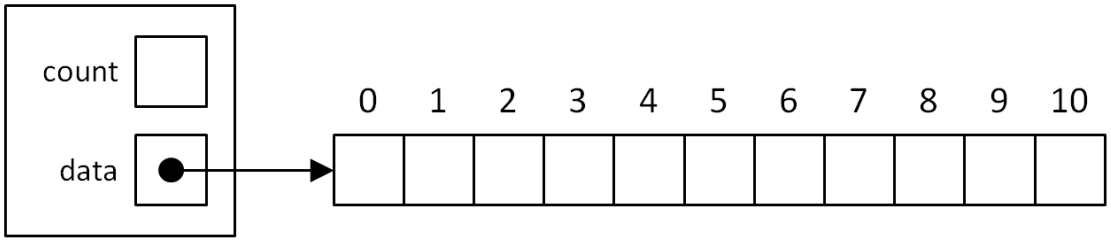



**(e)** [5 marks] Provide Pete with the values after one *poll()* operation.




Count = 9, 31, 22, 14, 18, 15, 12, 11, 6, 9

**(f)** [5 marks] Again using Pete's original state, provide him with the values after one *offer(***33**) operation.




Count = 11, 55, 33, 31, 18, 22, 14, 11, 6, 9, 12, 15,

**Question 9. Various other questions** [12 marks]

**(a)** [2 marks] It is possible to build and use linked lists by using no more than class LinkedNode. Why does it make sense to also provide a class LinkedList?

clients do not have to deal with null values isEmpty() supported can support "count" place for special cases natural place for invoking recursive methods on linked nodes.

**(b)** [2 marks] Your colleague Sally suggests replacing your company's current LinkedList-based implementation of a Set data structure with a BinarySearchTree-based implementation.

What speed-up in terms of asymptotic complexity should Sally expect for the operation "remove" in the average case?

$O(n) \rightarrow O(logn)$

**(c)** [2 marks] What speed-up in terms of asymptotic complexity should Sally expect for the operation "remove" in the worst-case and when would this worst-case occur?

$O(n)$ when the tree degenerates into a list.

**(d)** [2 marks] Suggest an even better solution to Sally.

HashSet or Balanced tree.

**(e)** [2 marks] What approach would you use to efficiently print the hundred largest numbers out of an unsorted collection of one billion elements?

Aborted SelectionSort or (better) HeapSort, or PriorityQueue.

**(f)** [2 marks] How would you determine whether two binary search trees contain the same values?

Use two in-order iterators or two sets.

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**SPARE PAGE FOR EXTRA ANSWERS**

Cross out rough working that you do not want marked.
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# Appendix (may be removed)

**Brief (and simplified) specifications of some relevant interfaces and classes.**

```
interface Collection<E>
    public boolean isEmpty()
    public int size()
    public boolean add(E item)
    public boolean contains(Object item)
    public boolean remove(Object element)
    public Iterator<E> iterator()

interface List<E> extends Collection<E>
    // Implementations: ArrayList, LinkedList
    public E get(int index)
    public E set(int index, E element)
    public void add(int index, E element)
    public E remove(int index)
    // plus methods inherited from Collection

interface Set extends Collection<E>
    // Implementations: ArraySet, HashSet, TreeSet
    // methods inherited from Collection

interface Queue<E> extends Collection<E>
    // Implementations: ArrayQueue, LinkedList, PriorityQueue
    public E peek () // returns null if queue is empty
    public E poll () // returns null if queue is empty
    public boolean offer (E element) // returns false if fails to add
    // plus methods inherited from Collection

class Stack<E> implements Collection<E>
    public E peek () // returns null if stack is empty
    public E pop () // returns null if stack is empty
    public E push (E element) // returns element being pushed
    // plus methods inherited from Collection

interface Map<K, V>
    // Implementations: HashMap, TreeMap, ArrayMap
    public V get(K key) // returns null if no such key
    public V put(K key, V value) // returns old value, or null
    public V remove(K key) // returns old value, or null
    public boolean containsKey(K key)
    public Set<K> keySet() // returns a Set of all the keys
```

```
interface Iterator<E>
    public boolean hasNext();
    public E next();
    public void remove();

interface Iterable<E>                  // Can use in the "for each" loop
    public Iterator<E> iterator();

interface Comparable<E>                // Can compare this to another E
    public int compareTo(E o);         // -ve if this less than o; +ve if greater than o;

interface Comparator<E>                // Can use this to compare two E's
    public int compare(E o1, E o2);    // -ve if o1 less than o2; +ve if greater than o2
```

---

```
class Collections
    public static void sort( List<E>)
    public static void sort( List<E>, Comparator<E>)
    public static void shuffle( List<E>, Comparator<E>)

class Arrays
     public static <E> void sort(E[ ] ar, Comparator<E> comp);

class Random
    public int nextInt( int n);    // return a random integer between 0 and n-1
    public double nextDouble(); // return a random double between 0.0 and 1.0

class String
    public int length()
    public String substring( int beginIndex, int endIndex)
```

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* * * * * * * * * * * * * * *