



## EXAMINATIONS — 2009

## MID YEAR

**COMP103**  
**Introduction to**  
**Data Structures and Algorithms**

**Time Allowed:** 3 Hours

- Instructions:**
1. Attempt **all** of the questions.
  2. *Read each question carefully before attempting it.* (Suggestion: You do not have to answer the questions in the order shown. Do the questions you find easiest first.)
  3. This examination will be marked out of **180** marks, so allocate approximately 1 minute per mark.
  4. Write your answers in the boxes in this test paper and hand in all sheets.
  5. Non-electronic translation dictionaries are permitted.
  6. Calculators are allowed.
  7. Documentation on relevant Java classes and interfaces is at the end of the paper.

<b>Questions</b>	<b>Marks</b>
1. Basic Questions	[20]
2. Using Collections	[24]
3. Implementing Array-based Collections	[16]
4. Linked Collections	[15]
5. Sorting	[20]
6. Trees	[30]
7. Partially Ordered Trees and Priority Queues	[25]
8. Hashing	[30]

**Question 1. Basic Questions**

[20 marks]

(a) [2 marks] Suppose we add items  $A$ ,  $B$  and  $C$  to an initially empty Set, then  $\text{remove}(C)$ ,  $\text{remove}(B)$ , and add  $C$  again. What is the contents of the Set?

(b) [2 marks] Suppose we add items  $A$ ,  $B$  and  $C$  to an empty stack, in that order, then remove two (using  $\text{pop}$ ), then add  $C$  again, then do one more  $\text{pop}$ . What will the next  $\text{pop}$  return?

(c) [2 marks] What is the asymptotic (“big-O”) cost of searching for an item in a Bag implemented using an *unsorted* array?

(d) [2 marks] Which specific interface do the items in a collection need to implement, in order for a programmer to be able to use the `for each` syntax (such as “`for (String str : wordCollection)`”) in Java?

(e) [2 marks] Name a fast ( $O(n \log n)$ ) sorting algorithm which is not “in-place”, that is, it requires a second array.

(f) [2 marks] What is the difference between a binary tree and a binary search tree?

**(g)** [2 marks] If a collection contains  $2^k$  elements and they are stored in a perfectly balanced binary search tree, what is the depth of the tree?

**(h)** [2 marks] If you use a stack to store nodes during an iterative tree traversal, will you get a breadth-first or depth-first traversal?

**(i)** [2 marks] What is the main drawback of using linear probing to resolve collisions in hashing?

**(j)** [2 marks] What is the defining property of a perfect hash function?

## Question 2. Using Collections

[24 marks]

For this question, assume a `PriorityQueue` implementation in which the head of the queue is the *largest* according to the natural ordering determined by `compareTo`.

(a) [5 marks] Suppose you have a large List of `Person` objects, where the `Person` class has a field `ageInYears` specifying the person's age. In words, describe how you could use a Priority Queue to efficiently find the 10 oldest people in the list.

Given a large text document, it might be useful to list all the unique words in the text together with how often each word occurs. Here is a class for representing words and their occurrences:

```
class WordFrequency implements Comparable <WordFrequency> {  
    String word;  
    int occurrences=0;  
    public WordFrequency(String w, int c) {  
        word = w;  
        occurrences=c;  
    }  
    public void incrementCount() {  
        occurrences += 1;  
    }  
    public int compareTo(WordFrequency other) {  
        ...  
    }  
}
```

(b) [3 marks]

Complete the `compareTo` method of the `WordFrequency` class.

```
public int compareTo(WordFrequency other) {  
  
  
  
  
  
  
  
  
  
}
```

(c) [8 marks] Complete the `countWordOccurrences` method that takes a scanner to a text file, and returns a map with words as the keys and `WordFrequency` objects as the values.

```
public Map <String, WordFrequency> countWordOccurrences(Scanner sc) {
```

```
}
```

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Cross out rough working that you do not want marked.  
Specify the question number for work that you do want marked.

(d) [8 marks]

Complete the method `printMostCommonWords` that takes the above map as an argument and prints out the 100 most common words in it.

```
public void printCommonWords(Map <String, WordFrequency> wordmap) {
```

```
}
```

### Question 3. Implementing Array-based Collections

[16 marks]

The following gives an implementation of `ArrayQueue`, which inserts at the end of an array if there is space, and moves all the elements in the array down when it polls.

```
public class ArrayQueue <E> implements Queue <E> {
    private E [ ] data;
    private int count;
    private static final int INITIALCAPACITY = 16;

    public ArrayQueue() {
        data = (E [ ]) (new Object[INITIALCAPACITY]);
    }
    :
    :
    public boolean offer (E item) {
        if (count == data.length) return false;
        data[count] = item;
        count++;
        return true;
    }

    public E poll () {
        if (count == 0) return null;
        E ans = data[0];
        for (int i=0; i < count-1; i++)
            data[i] = data[i+1];
        count = count-1;
        return ans;
    }
}
```

(a) [2 marks] What is the *average*-case asymptotic (“big-O”) cost of offer?

(b) [2 marks] What is the *average*-case cost of poll?



The `ArrayQueue` class shown on the previous page uses an array of fixed size. This means it will not be able to store more elements than the size of the array. Suppose we replace the `offer` method with the following instead, which uses a `doubleAndCopy` method:

```
public boolean offer (E item) {  
    if (count == data.length) doubleAndCopy();  
    data[count] = item;  
    count++;  
    return true;  
}
```

(c) [5 marks] Complete the `doubleAndCopy()` method, which doubles the size of the array when the queue is full.

```
private void doubleAndCopy() {
```

```
}
```

This `ArrayQueue` implementation on the previous page is also inefficient, because every time an element is removed by a `poll()` it moves all the remaining elements down. Suppose, instead, we avoid having to move the elements down by storing the location of the first element of the queue in a new variable.

(d) [2 marks] If we do this, what is the *average*-case cost (in “big-O” notation) of `poll`?

(e) [5 marks] What is the problem with this idea? Describe a strategy for solving this problem which has the same cost, but avoids the problem you identified.



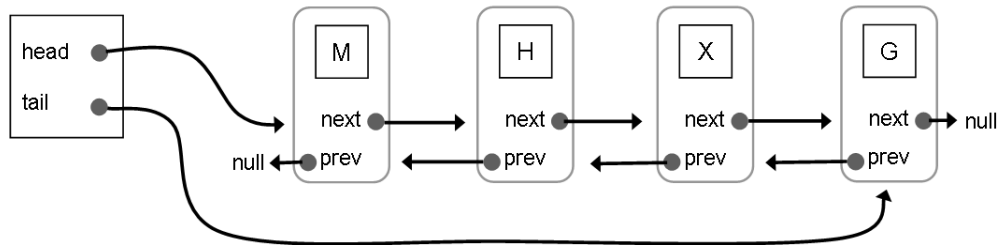
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Cross out rough working that you do not want marked.  
Specify the question number for work that you do want marked.

## Question 4. Linked Collections

[15 marks]

In lectures we discussed linked lists. The following diagram shows an example of a “doubly linked” list, in which each node has a link both to the next node in the list as usual, but it also has a link to the previous node. Additionally, the header node has links to both ends of the list in this version:



The code below gives an implementation of a doubly linked list, and lists some of its methods:

```
public class DoublyLinkedList <E> implements AbstractList <E> {  
    private ListNode<E> head = null;  
    private ListNode<E> tail = null;  
    public DoublyLinkedList(){} // constructor  
    public int size () {...  
    public boolean isEmpty(){...  
    public void add(E item){...  
    public boolean remove(E element){...  
    public Iterator <E> iterator () {...
```

DoublyLinkedList uses the ListNode class:

```
public class ListNode <E> {  
    private E value;  
    private ListNode<E> next, prev;  
    public ListNode(E item, ListNode<E> nextNode, ListNode<E> prevNode ){  
        value = item;  
        next = nextNode;  
        prev = prevNode;  
    }  
    public E get() { return value; }  
    public ListNode<E> next() { return next; }  
    public void set(E item) { value = item; }  
    public void setNext(ListNode<E> nextNode) { next = nextNode; }  
    public void setPrev(ListNode<E> prevNode) { prev = prevNode; }  
    :  
}
```



**Question 5. Sorting**

[20 marks]

Suppose we ran a sorting algorithm on the following array (assume left-to-right, A-Z):

B	D	F	J	E	H	A	C	I	G
---	---	---	---	---	---	---	---	---	---

(a) [2 marks] Which elements would selection sort swap first?

(b) [2 marks] Which elements would insertion sort swap first?

(c) [8 marks] The “fast” sorting algorithms we met in COMP103 were MergeSort, QuickSort, TreeSort and HeapSort. Which two of these have *worst-case* costs that scale as  $O(n^2)$ ?

Give the algorithm name, and the reason (or describe what the worst case is for this algorithm).

Algorithm:  
Reason:

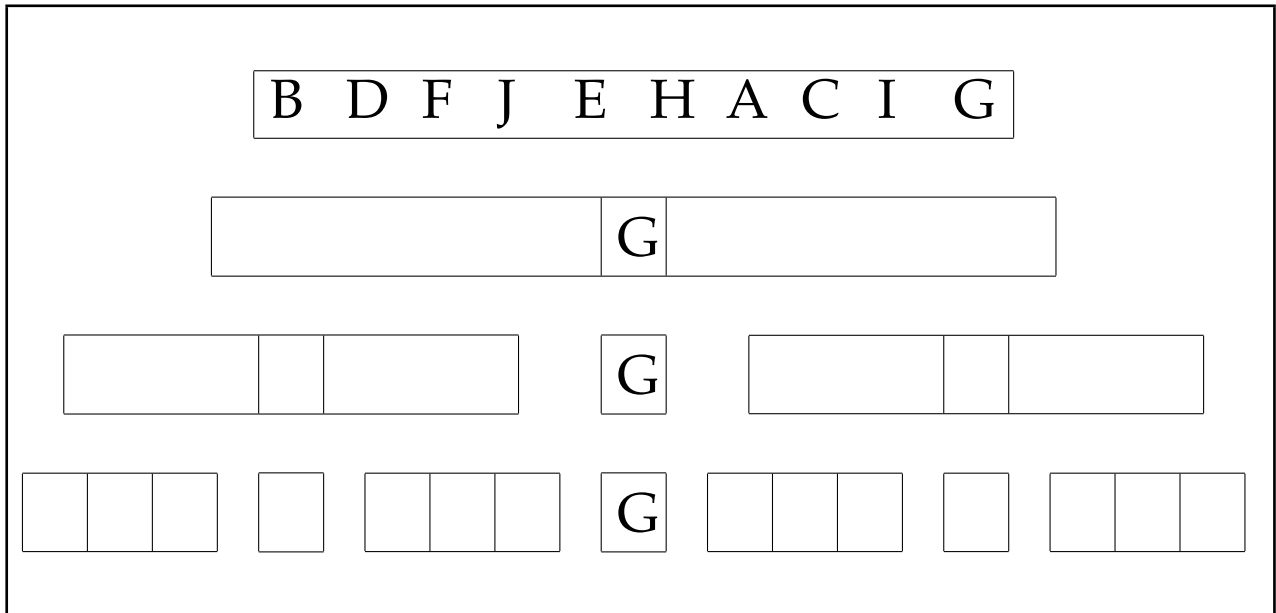
  
  
  
  
  
  
  
  
  
  

Algorithm:  
Reason:

The diagram below represents QuickSort sorting an array. Complete the diagram to show the elements in each sub-array as the algorithm progresses. Suppose our pivot algorithm always chooses the right-most element of the sub-array as the pivot (the first one is done for you).

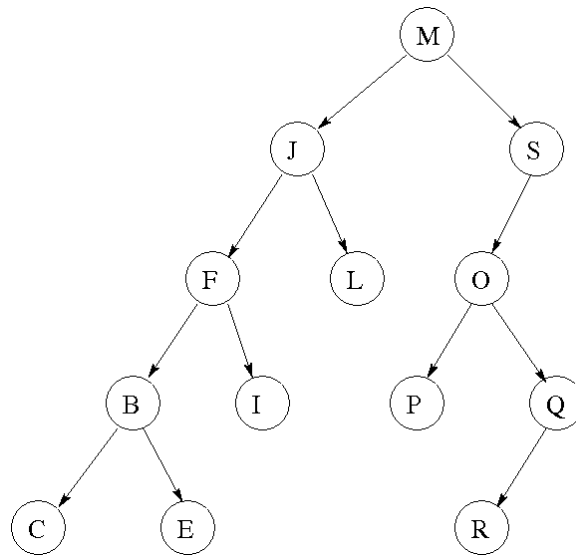
**Hint! boxes may contain different numbers of elements.**

**(d)** [8 marks] Complete the diagram showing how QuickSort progresses at each step.



**Question 6. Trees**

[30 marks]



**(a)** [2 marks] What is the depth of this tree?

**(b)** [2 marks] What is the minimum depth that a tree with the same elements could have?

**(c)** [4 marks] The tree shown above is almost, but not entirely a Binary Search Tree. Modify the tree to be a correct Binary Search Tree with the same elements.



The following algorithm traverses a tree and prints out all of the nodes in the tree using a stack.

```
public static void print (BinaryTreeNode tree) {  
    Stack<BinaryTreeNode> stack = new Stack<BinaryTreeNode>();  
    stack.push(tree);  
    while (!stack.isEmpty()) {  
        BinaryTreeNode node = stack.pop();  
        System.out.println(node.value);  
        if (node.right != null) stack.push(node.right);  
        if (node.left != null) stack.push(node.left);  
    }  
}
```

(d) [6 marks] Show the state of the stack when 'B' is printed if this algorithm is run on the (unmodified) tree at the beginning of this question.

Question 6(d) traversed a binary tree which has *left* and *right* child nodes. A general tree can have any number of child nodes, so they are often stored in a list. The following code implements a `GeneralTreeNode` class.

```
class GeneralTreeNode<E> {
    List<GeneralTreeNode> children = new ArrayList<GeneralTreeNode>();
    E value;
    public GeneralTreeNode(E value) {
        this.value = value;
    }
}
```

(e) [6 marks] Complete this `printNodesWithTwoChildren()` method which **recursively** prints out the value of all nodes in the tree which have two children.

You may use any recursive traversal algorithm.

```
public static void printNodesWithTwoChildren(GeneralTreeNode<E> node) {
```

```
}
```

Suppose you are given this ordered sequence of values stored in an array:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

If you inserted these nodes into a Binary Search Tree in this order then the resulting tree would be completely unbalanced: each element would be added as a child of the previous one, so you would end up with a linked list instead of a tree!

(f) [5 marks] Give (draw) a re-ordering of these elements that would produce a balanced Binary Search Tree.

**Hint: Drawing the tree first might help you!**

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

(g) [5 marks] (*Hard*) Given an *arbitrary* (unsorted) ordered sequence of nodes, explain how you could produce a balanced Binary Search Tree.

**Hint: do not write Java code**

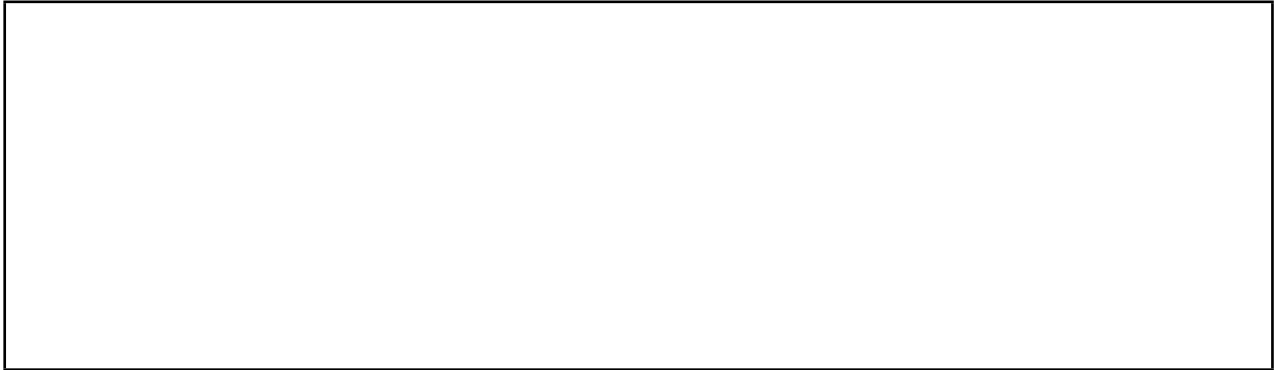
**Question 7. Partially Ordered Trees and Priority Queues**

[25 marks]

The following array represents a heap (partially ordered tree).

B	F	D	L	O	E	K	N	P	T
---	---	---	---	---	---	---	---	---	---

(a) [3 marks] Draw the heap as a tree.



(b) [3 marks] Redraw the tree after the following operations: Add 'R', Add 'C'.



(c) [3 marks] Redraw the tree from your previous answer after removing an element (poll).



In lectures we discussed implementing a Partially Ordered Tree (POT) as an Array. It is also possible to implement a POT using linked nodes. The following two classes show the fields (but not the methods) of an implementation of a POT using linked nodes:

```

class POT<E extends Comparable> {
    POTNode tree; //top node in the tree
    POTNode last; //last node in the tree
    ...
}

class POTNode<E extends Comparable> {
    E value;
    POTNode parent; //parent node
    POTNode left; //left child
    POTNode right; //right child
    POTNode previous; //previous node in the tree
    ...
}

```

The rest of this question considers different parts of this implementation.

Removing (polling) a value from a partially ordered tree in an array requires removing the element at the front, moving the last element into its place, then calling "pushdown" to find the correct place for the element you moved. It is similar for a POT implemented with linked nodes.

```

//these methods belong to the POT class
public E poll () {
    if (tree == null) return null; //the POT is empty
    E value = tree.value;

    //move the value from the last node to the top of the tree
    tree.value = last.value;

    //remove the last node
    if (last.parent != null) {
        if (last.parent.left == last) last.parent.left = null;
        else last.parent.right = null;
    }
    if (last.previous != null) last = last.previous;
    else { //tree is now empty
        last = null;
        tree = null;
    }

    //push down the value in the top node to its correct place
    if (tree != null) pushdown(tree);
}

```

(d) [6 marks] Implement the pushdown method for the POT class:

```
public void pushdown(POTNode<E> node) {
```

```
}
```

Adding an element to this POT implementation is slightly more complex. We need to add a new node at the first available place in the POT (at the end), then push the value up to its correct place in the tree.

Here is a *pseudo-code* description of the method for adding an element:

- create a new node, and set its value to the new value
- find the first node in the tree which does not have two children, store as parent of our node
- find the last node in the tree, store as previous of our node
- update the last pointer to point to our node
- set the left (or right if the parent has a child) field in the parent to point to our node
- perform "pushup" on our node

(e) [3 marks] This description does not include any error checks.

Are there any situations where an error could occur, and if so what should the method do to resolve them?

(f) [3 marks] How could we find the first available place in the tree? (Step 2 in the method)

(g) [4 marks] Why is a node-based implementation a bad choice for implementing a partially ordered tree?



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Cross out rough working that you do not want marked.  
Specify the question number for work that you do want marked.

**Question 8. Hashing**

[30 marks]

(a) [3 marks] With probing in Hash Tables, why is it a good idea to rehash to a larger table well before the current one is full?

(b) [4 marks] Explain why iterating over the items stored in a HashSet can be a slow process.

(c) [5 marks] Once a Hash Table gets too full, it needs to move the items to an larger array. Why is doubling and copying the array elements (as we did for `ArrayList` for example) *not* the correct way to do this, and what is the correct way?

(d) [4 marks] When writing an implementation of a HashSet and using open addressing (probing), the operation `add` is easy to achieve but `remove` requires more thought. Why is `remove` more challenging?

(e) [4 marks] Why is it important that two items that are `“.equal()”` have the same `hashCode`?

(f) [10 marks]

Suppose you want to write a program to store information about your collection of music CDs. You decide to use a `HashSet`, and you write a class `MusicCD`, with the fields shown below.

A `MusicCD` is identified by its `artist`, `albumTitle`, and `dateBought` fields.

```
public class MusicCD{
    private final String artist ;
    private final String albumTitle;
    private final Date dateBought;

    private String whoBorrowed; // may change
    private String whereStored; // may change
    private int myRating; // a number between 1 and 5, may change
    :
```

For your own purposes you also want to keep track of `whoBorrowed` (which could be null), `whereStored` (e.g. "under my bed") and `myRating` information. These three fields may change over time, but don't affect the identity of the CD itself.

Complete the following `hashCode` and `equals` methods for the `MusicCD` class. You may assume that the `String` and `Date` classes have appropriate `hashCode` functions.

```
public int hashCode(){
```

```
}
```

```
public boolean equals(Object obj){
```

```
}
```

**SPARE PAGE FOR EXTRA ANSWERS**

Cross out rough working that you do not want marked.  
Specify the question number for work that you do want marked.

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## Appendices

### Possibly useful formulas:

- $1 + 2 + 3 + 4 + \dots + k = \frac{k(k+1)}{2}$
- $1 + 2 + 4 + 8 + \dots + 2^k = 2^{k+1} - 1$
- $a + (a + b) + (a + 2b) + \dots + (a + kb) = \frac{(2a+kb)(k+1)}{2}$
- $a + as + as^2 + as^3 + \dots + as^k = \frac{as^{k+1}-a}{s-1}$

### Table of base 2 logarithms:

$n$	1	2	4	8	16	32	64	128	256	512	1024	1,048,576
$\log_2(n)$	0	1	2	3	4	5	6	7	8	9	10	20

### Brief (and simplified) specifications of relevant interfaces and classes.

#### public class Random

```
public int nextInt(int n);           // return a random integer between 0 and n-1
public double nextDouble();         // return a random double between 0.0 and 1.0
```

#### public interface Iterator <E>

```
public boolean hasNext();
public E next();
public void remove();
```

#### public interface Iterable <E>

```
public Iterator <E> iterator(); // Can use in the "for each" loop
```

#### public interface Comparable <E>

```
public int compareTo(E o); // Can compare this to another E
```

#### public interface Comparator <E>

```
public int compare(E o1, E o2); // Can use this to compare two E's
```

#### DrawingCanvas class:

```
public void drawLine(int x, int y, int u, int v) // Draws line from (x, y) to (u, v)
public void drawOval(int x, int y, int wd, int ht) // Draws outline of oval
public void drawString(String str, int x, int y) // Prints str at (x, y)
```

```

public interface Collection<E>
    public boolean isEmpty();
    public int size ();
    public boolean contains(Object item);
    public boolean add(E item);           // returns false if failed to add item
    public Iterator <E> iterator();

```

```

public interface List<E> extends Collection<E>
    // Implementations: ArrayList
    public E get(int index);
    public void set(int index, E element);
    public void add(int index, E element);
    public void remove(int index);
    public void remove(Object element);

```

```

public interface Set extends Collection<E>
    // Implementations: ArraySet, SortedArraySet, HashSet
    public boolean contains(Object element);
    public boolean add(E element);
    public boolean remove(Object element);

```

```

public interface Queue<E> extends Collection<E>
    // Implementations: ArrayQueue, LinkedList
    public E peek ();           // returns null if queue is empty
    public E poll ();          // returns null if queue is empty
    public boolean offer (E element);

```

```

public class Stack<E> implements Collection<E>
    public E peek ();           // returns null if stack is empty
    public E pop ();           // returns null if stack is empty
    public E push (E element); // returns element

```

```

public interface Map<K, V>
    // Implementations: HashMap, TreeMap, ArrayMap
    public V get(K key);           // returns null if no such key
    public V put(K key, V value); // returns old value, or null
    public V remove(K key);       // returns value removed, or null
    public boolean containsKey(K key);
    public Set<K> keySet();        // returns set of all keys in Map
    public Collection<V> values(); // returns collection of all values
    public Set<Map.Entry<K, V>> entrySet(); // returns set of (key–value) pairs

```

**Scanner class:**

```

public boolean hasNext()           // Returns true if there is more to read
public boolean hasNextInt()        // Returns true if the next token is an integer
public String next()               // Returns the next token (chars up to a space/line)
public String nextLine()           // Returns string of chars up to next newline
public int nextInt ()              // Returns the integer value of the next token

```