TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

VICTORIA UNIVERSITY OF WELLINGTON

Student ID: ....

EXAMINATIONS — 2009

END YEAR

COMP103
Introduction to
Data Structures and Algorithms

Time Allowed: 3 Hours

Instructions:

1. Attempt **all** of the questions.
2. *Read each question carefully before attempting it.* (Suggestion: You do not have to answer the questions in the order shown. Do the questions you find easiest first.)
3. This examination will be marked out of **180** marks, so allocate approximately one minute per mark.
4. Write your answers in the boxes in this test paper and hand in all sheets.
5. Non-electronic translation dictionaries are permitted.
6. Calculators are allowed.
7. Documentation on relevant Java classes and interfaces is at the end of the paper.

Questions	Marks
1. Basic Questions	[20]
2. Using Collections	[26]
3. Implementing Collections	[15]
4. Linked Structures	[24]
5. Trees and Graphs	[40]
6. Binary Search Trees	[20]
7. Partially Ordered Trees and Heaps	[35]

Question 1. Basic Questions [20 marks]

Apart from Bag and Map, the basic collection types we have looked at are

- Set
- List
- Stack
- Queue

(a) [2 marks] Which of the above types restrict access to the collection?

(b) [2 marks] Which of the above types allow duplicates in the collection?

(c) [2 marks] What is the *best case* "big-O" cost of insertion sort, on an array of n items?

(d) [2 marks] What is the *worst case* "big-O" cost of insertion sort, on an array of n items?

(e) [4 marks] Name TWO sorting algorithms whose average case "big O" cost is $O(n \log n)$ but whose worst case is $O(n^2)$.

(Question 1 continued on next page)

(Question 1 continued)

(f) [2 marks] Give TWO properties that are desirable to have in a hash function.

- property:

- property:

(g) [4 marks] Why is it important to ensure that a hash table does not get too full?

(h) [2 marks] What happens to a Java object when it is no longer referenced by any other objects?

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 2. Using collections [26 marks]

(a) [6 marks] Suppose that you want to determine whether two Sets of Strings are equal, *i.e.,* contain the same elements. Since a Set does not impose any ordering on its elements, you cannot just iterate through the two sets, comparing the elements, because the elements might have been stored in different orders.

Complete the equalSets method below. Your method should work independently of how the Sets of Strings are implemented.

```
public static boolean equalSets(Set<String> set1, Set<String> set2) {




}
```

(Question 2 continued on next page)

A local trust owns a small hall, which they make available to community groups in the evenings. They would like a simple system for keeping track of reservations for use of the hall. Only one group can have use of the hall on a given evening. The program consists of a class BookingSystem, which begins by initialising a map as follows:

```
public class BookingSystem {
  private Map <Date, String> bookings = new HashMap<Date, String>();
```

The class now needs to have three methods, for making, checking, and summarizing bookings.

(b) [5 marks]

Complete the checkBooking method, which takes a date, and either prints out who has reserved the hall on that date, or prints out that there is no booking for that date.

```
  public void checkBooking(Date date) {

  }
```

(c) [5 marks] Complete the makeBooking method, which takes a date and name, and either makes the booking and prints out that the booking has been made, or prints out that the date has already been booked. Note that Data has a toString method.

```
  public void makeBooking(Date date, String name) {

  }
```

(Question 2 continued on next page)

(Question 2 continued)

(d) [10 marks] Complete the bookingSummary method, which prints out a summary of who has bookings in the system. It should print out the name of each group with bookings in the system, and along with the number of bookings that group has in the system. Note that each group's name should only be printed once.

```
  public void bookingSummary() {

}
```

Question 3. Implementing collections [15 marks]

For this question, you are to complete some of the methods of an ArrayQueue class, which implements a queue using an array. In this implementation, *the head of the queue is to be the first element of the array*, and count is the number of elements in the queue.

The header and fields of the class are given below:

```
public class ArrayQueue <E> extends AbstractQueue<E> {
  private static int INITIALCAPACITY = 8;
  private int count = 0;
  private E[ ] data;
```

(a) [3 marks] Complete the following constructor, which should just initialise the data field.

```
public ArrayQueue() {

}
```

(b) [2 marks] Complete the isEmpty method, which returns true if and only if the queue is empty.

```
  public boolean isEmpty() {

  }
```

(c) [3 marks] Complete the following offer method, which adds a value onto the end of the queue. Assume that ArrayQueue has an ensureCapacity() method, which "doubles and copies" the data array if the current one is full.

```
public void offer(E item) {

}
```

(Question 3 continued on next page)

(Question 3 continued)

(d) [7 marks] Complete the following poll method which removes and returns the value at the head of the queue if the queue is not empty, and otherwise throws an EmptyQueueException.

```
public E poll () {

}
```

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Question 4. Linked Structures [24 marks]

The following class can be used to represent a simple linked list:

```
public class LinkedNode <E> {
    private E value;
    private LinkedNode<E> next;
    public LinkedNode(E item, LinkedNode<E> nextNode) {
        value = item;
        next = nextNode;
    }
    public E get()   { return value;   }
    public LinkedNode<E> next() {
        return next;
    }
    public void set(E item) {
        value = item;
    }
    public void setNext(LinkedNode<E> nextNode) {
        next = nextNode;
    }
}
```

(a) [6 marks] Complete the length method, which takes a LinkedNode as an argument and returns the length of the list starting at that node.

```
public int length(LinkedNode n) {



}
```

(b) [8 marks] Complete the append method, which takes two LinkedNode arguments. Each of these is assumed to be the start of a separate linked list (i.e. the two lists have no nodes in common). The append method returns a list containing all of the nodes of the first list followed by all of the nodes of the second list. If the first list is non-empty, its last node should be modified to point to the first node of the second list; if the first list is empty, append should return the second list.

```
public LinkedNode append(LinkedNode l1, LinkedNode l2) {

}
```

(c) [4 marks] What is the "big-O" cost of your append method, as a function of the lengths, n_1 and n_2, of the two lists?

(d) [6 marks] Explain how the representation of linked lists can be modified so that the append method is more efficient.

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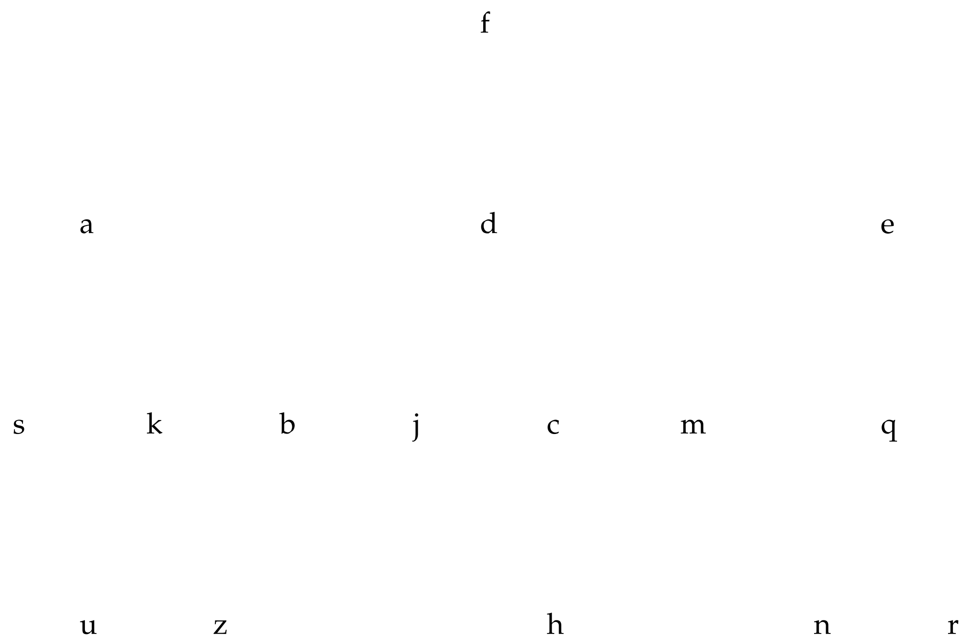
Question 5. Trees and Graphs

[40 marks]

(a) [8 marks] Consider the following tree:



(i) [1 mark] Which node is the *root* of the tree?

(ii) [2 marks] How many *leaves* does the tree have?

(iii) [1 mark] Which node is the *parent* of node *q*?

(iv) [2 marks] Which nodes are the *children* of node *a*?

(v) [2 marks] Which nodes has the *most children*?

(b) [15 marks]

The *fringe* of a tree is the sequence obtained by collecting up the labels on the leaves in the order they are visited during a left-to-right depth first traversal.

(i) [3 marks] What is the fringe of the tree shown in part **(a)** above?

(ii) [12 marks] Complete the printFringe method, which takes a tree as an argument and prints out its fringe.

You should assume that the Tree class has methods:

```
public String label();              // Returns the label at the root
public List<Tree> subtrees();       // Returns the list of subtrees
```

```
public void printFringe(Tree t) {

}
```

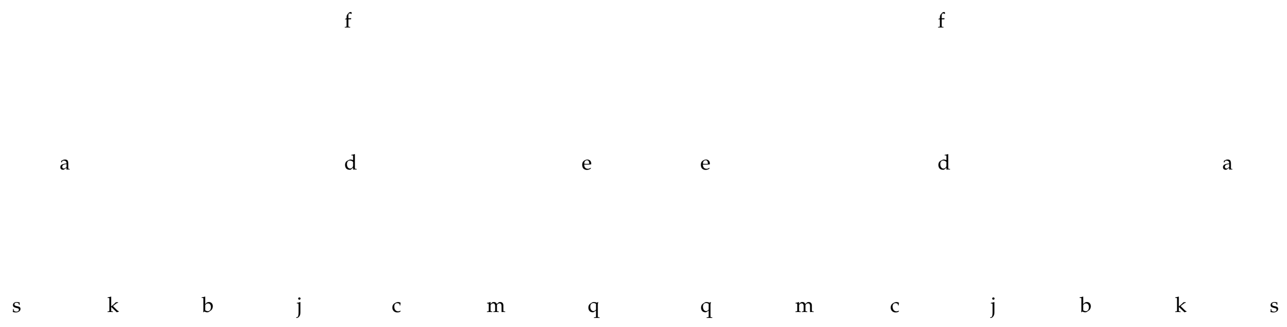
(c) [5 marks] Suppose you now want to print the fringe of a directed *graph*, where a node is on the fringe if it has no successors (outward edges). Assume that the Tree class is replaced by a Graph class with a successors method that returns a list of successors in place of the subtrees method.

What additional change(s) would you need to make to the printFringe method?

(d) [12 marks]

The *reflection* of a tree is the mirror image of the given tree, and can be constructed by recursively reversing the list of subtrees of every node.

For example, the following diagram shows a tree and its reflection.



Complete the reflect method, which takes a tree as an argument and returns its reflection.

You should assume that the Tree class has the following constructor, which returns a tree with l at its root and subtrees s:

```
public Tree(String l, List<Tree> s)
```

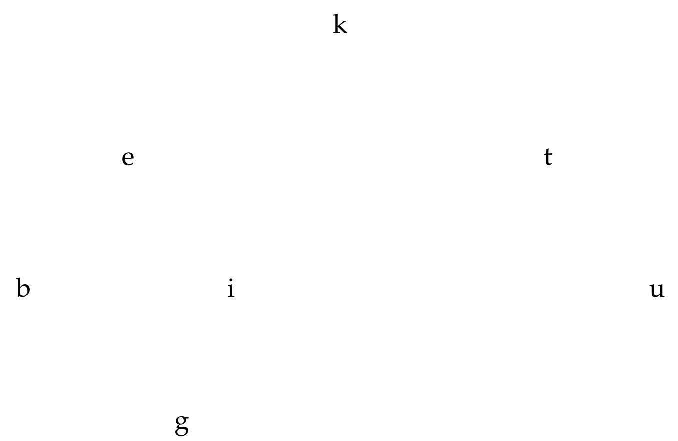
```
public Tree reflect (Tree t) {

}
```

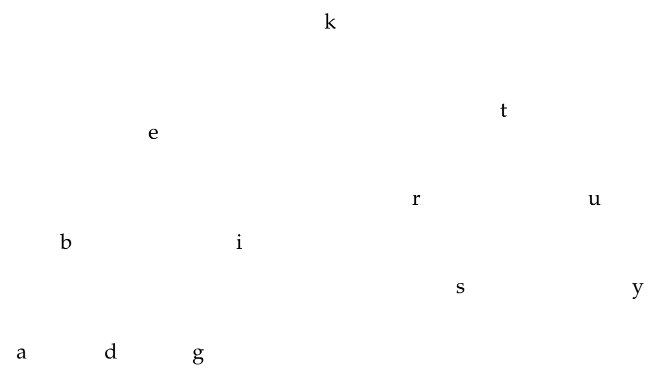
Question 6. Binary Search Trees [20 marks]

(a) [8 marks] What ordering property must be satisfied by the labels of a Binary Search Tree?

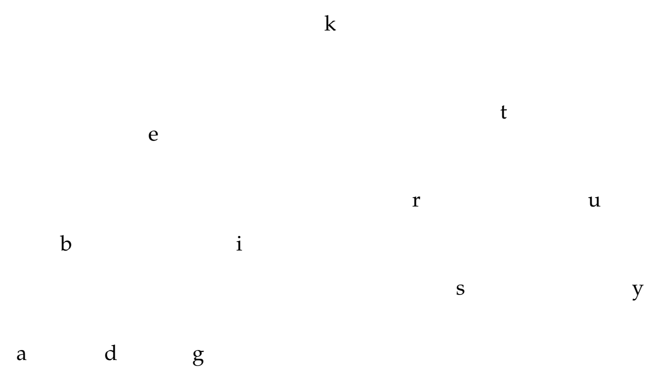
(b) [4 marks] Show the effect of adding the values *j*, *c*, *r*, *s* (in that order) to the following Binary Search Tree.



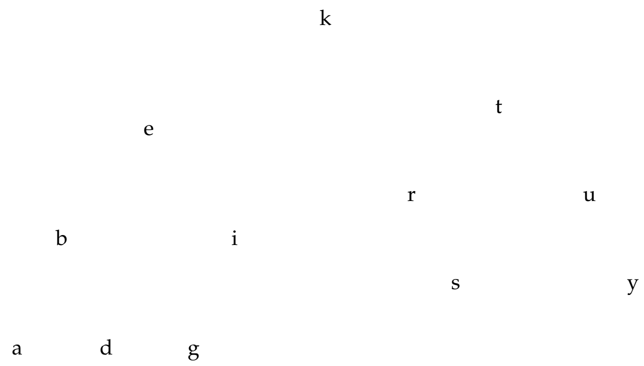
(c) [2 marks] Show the effect of deleting *d* from the following Binary Search Tree.



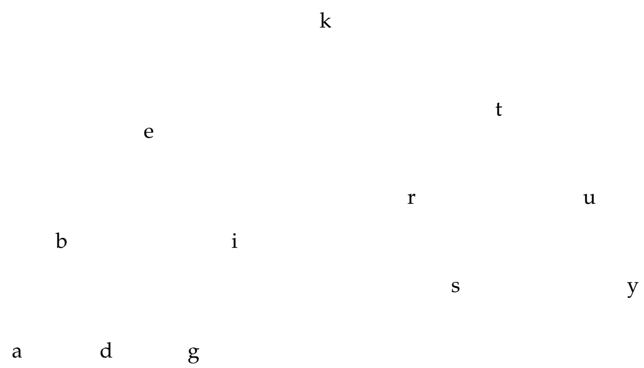
(d) [2 marks] Show the effect of deleting *i* from the following Binary Search Tree.



(e) [2 marks] Show the effect of deleting *t* from the following Binary Search Tree.



(f) [2 marks] Show the effect of deleting *k* from the following Binary Search Tree.



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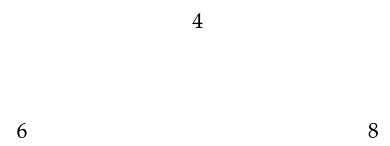
Question 7. Partially Ordered Trees and Heaps [35 marks]

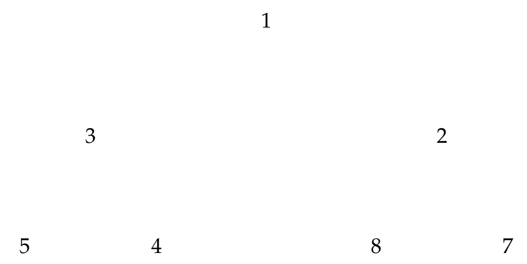
A Partially Ordered Tree is a binary tree used to represent a set in way that allows fast implementations of operations to add an element and to remove the *smallest* element of the set.

(a) [6 marks] What ordering property must be satisfied by the labels of a Partially Ordered Tree?

(b) [6 marks] Show the effect of adding values 5, 3 and 2 (in that order) to the following Partially Ordered Tree. You should show the tree resulting after each insertion.



(c) [6 marks] Show the effect of removing the smallest value **three** times starting with the following Partially Ordered Tree. You should show the tree resulting after each removal.



(d) [17 marks] A *heap* is an array implementation of a complete Partially Ordered Tree. In a heap:

(i) [3 marks] What is the index of the *parent* of the node with index k?

(ii) [4 marks] What are the indexes of *children* of the node with index k?

(iii) [10 marks] Suppose you want to be able to remove the *largest* element of the set, as well as the *smallest*. Explain how you can find and remove the largest element in a set represented as a heap, and give the cost of this operation.

****

Appendices

Possibly useful formulae:

- $1 + 2 + 3 + 4 + \cdots + k \;=\; \frac{k(k+1)}{2}$
- $1 + 2 + 4 + 8 + \cdots + 2^k \;=\; 2^{k+1} - 1$

Table of base 2 logarithms:

n	1	2	4	8	16	32	64	128	256	512	1024	1,048,576
$\log_2(n)$	0	1	2	3	4	5	6	7	8	9	10	20

Brief (and simplified) specifications of relevant interfaces and classes.

```
public class Random
   public int nextInt ( int n);            // return a random integer between 0 and n−1
   public double nextDouble();             // return a random double between 0.0 and 1.0

public interface Iterator <E>
   public boolean hasNext();
   public E next ();
   public void remove();

public interface Iterable <E>              // Can use in the "for each" loop
   public Iterator <E> iterator ();

public interface Comparable<E>             // Can compare this to another E
   public int compareTo(E o);

public interface Comparator<E>             // Can use this to compare two E's
   public int compare(E o1, E o2);

DrawingCanvas class:
   public void drawLine(int x, int y, int u, int v)      // Draws line from (x, y) to (u, v)
   public void drawOval(int x, int y, int wd, int ht)    // Draws outline of oval
   public void drawString(String str , int x, int y)     // Prints str at (x, y)
```

```
public interface Collection<E>
    public boolean isEmpty();
    public int size ();
    public boolean contains(Object item);
    public boolean add(E item);                  // returns false if failed to add item
    public Iterator <E> iterator();

public interface List<E> extends Collection<E>
    // Implementations: ArrayList
    public E get(int index);
    public void set(int index, E element);
    public void add(int index, E element);
    public void remove(int index);
    public void remove(Object element);

public interface Set extends Collection<E>
    // Implementations: ArraySet, SortedArraySet, HashSet
    public boolean contains(Object element);
    public boolean add(E element);
    public boolean remove(Object element);

public interface Queue<E> extends Collection<E>
    // Implementations: ArrayQueue, LinkedList
    public E peek ();                     // returns null if queue is empty
    public E poll ();                     // returns null if queue is empty
    public boolean offer (E element);

public class Stack<E> implements Collection<E>
    public E peek ();                     // returns null if stack is empty
    public E pop ();                      // returns null if stack is empty
    public E push (E element);            // returns element

public interface Map<K, V>
    // Implementations: HashMap, TreeMap, ArrayMap
    public V get(K key);                  // returns null if no such key
    public V put(K key, V value);         // returns old value, or null
    public V remove(K key);               // returns value removed, or null
    public boolean containsKey(K key);
    public Set<K> keySet();               // returns set of all keys in Map
    public Collection<V> values();        // returns collection of all values
    public Set<Map.Entry<K, V>> entrySet(); // returns set of (key-value) pairs

Scanner class:
    public boolean hasNext()              // Returns true if there is more to read
    public boolean hasNextInt()           // Returns true if the next token is an integer
    public String next()                  // Returns the next token (chars up to a space/line)
    public String nextLine()              // Returns string of chars up to next newline
    public int nextInt()                  // Returns the integer value of the next token
```