TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

Student ID: .....

EXAMINATIONS — 2012

END-OF-YEAR

COMP 103
INTRODUCTION TO DATA STRUCTURES AND ALGORITHMS

Time Allowed: 3 Hours ******** WITH SOLUTIONS **********

Instructions: Attempt ALL Questions.

Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found.

The exam will be marked out of 180 marks.

Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted.

Non-electronic foreign language dictionaries are permitted. Documentation on some relevant Java classes, interfaces, and exceptions can be found at the end of the paper.

There are spare pages for your working and your answers in this exam, but you may ask for additional paper if you need it.

Questions	**Marks**
1. Collections and Sorting	[15]
2. Using Priority Queues	[18]
3. Using Maps	[22]
4. Implementing a Collection	[15]
5. Binary Search	[15]
6. Trees	[25]
7. Linked Lists	[25]
8. Tree Traversals	[13]
9. Hashing	[18]
10. Debugging Tree Code	[14]

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 1. Collections and Sorting [15 marks]

(a) [3 marks] What is the difference between a priority queue and an ordinary queue?

An ordinary queue produces items strictly in order of arrival (first in, first out); a priority queue produces items according to their priority, so that poll always returns the highest priority item on the queue, regardless of when it was added to the queue.

(b) [3 marks] What are the differences between a Set and a List?

A set cannot have duplicates, and there is no order on the items; a list allows duplicates and keeps items in some order such that you can access items by their position in the order.

(c) [3 marks] If you need to sort a large collection of items, what is one reason why you might choose NOT to use *QuickSort*.

QuickSort has a bad worst case cost ($O(n^2)$). *or* QuickSort is not stable. or There are more efficient algorithms if the collection is almost sorted.

(d) [3 marks] HeapSort and MergeSort are both efficient sorting algorithms. Explain one advantage of HeapSort over MergeSort.

HeapSort is an in-place sorting algorithm - it doesn't require the additional space that MergeSort needs

(e) [3 marks] You could use an ArrayList to store a set of items, rather than some implementation of Set. Explain why it is generally better to use a Set class rather than ArrayList if you are storing a set of items.

The Set classes impose constraints on the collection (such as no duplicates) that you would have to implement yourself for an ArraySet. It is therefore easier and less error-prone to use a Set class. Also, the Set classes can be much more efficient than an arraylist for finding, adding, and removing items.

Question 2. Using Priority Queues [18 marks]

Suppose you are writing a program for the reception desk in a hospital emergency room. When patients arrive, the nurse must collect details from the patient (name, etc) and assess the urgency for treatment on a scale of 0 to 5. If the urgency is 5, the patient is sent straight through to the treatment area. Other patients must wait, and their details are put on a priority queue. Whenever a doctor becomes available, the nurse will remove the patient details from the front of queue, print them off, and send the patient through to the treatment area.

Assume that one field of your EmergencyRoom program is the priority queue:

```
private PriorityQueue<Patient> waitingQueue = new PriorityQueue<Patient>();
```

(a) [8 marks] Complete the following acceptNewPatient and sendNextPatient methods that perform the functions described above.

```
public void acceptNewPatient(){
    Patient patient = new Patient();
    patient.askDetails();
    int urgency = UI.askInt("Urgency of patient");
    patient.setUrgency(urgency);

    if (. . urgency == 5 . . . . . ){
        patient.printDetails();
        UI.println("Send patient through");
    }
    else {
        . . waitingQueue.offer(patient); . . . . . .
    }
}


public void sendNextPatient(){

    if (. . waitingQueue.isEmpty() . . . . . .
        UI.println("No patients waiting!!!");
    }
    else {
        Patient patient = . . waitingQueue.poll() . . . . . .
        patient.printDetails();
    }
}
```

(Question 2 continued on next page)

(Question 2 continued)

(b) [5 marks] For the priority queue to work correctly, Patient objects must be Comparable so that higher priority patients (eg urgency: 4) are ordered before lower priority patients (eg urgency: 1). Add whatever is required to the Patient class below to ensure that Patient objects are Comparable. (You do not need to complete the askDetails and printDetails methods.)

```
class Patient {
    private String name;
    private int age;
    private int urgency;

    public int compareTo(Patient other){
        return other.urgency - this.urgency;
    }
    public void setUrgency(int u){ urgency = u;  }
    public void askDetails (){...}
    // or
    public void printDetails (){...}
}
```

(c) [5 marks] Describe how you would modify the EmergencyRoom class and the Patient class (including your answer to part (b)) so that the priority queue not only ensured that higher urgency patients are treated before lower urgency patients, but also that patients who arrived earlier are treated before equal urgency patients who arrived later. Note: the PriorityQueue Class is part of the Java Collections and cannot be modified.

I would add an arrivalTime field to the Patient class, and make the Patient constructor set the time to System.currentTimeMillis() (or make acceptNewPatient set the arrival time when it sets the urgency). I would then make the compareTo method compare the times of the two patients if they have the same urgency, so that the patient with the earlier time comes first.
Alternatively, I would replace the PriorityQueue by an array of five ordinary queues (eg LinkdList), one for each urgency. I would add new patients to the queue corresponding to their urgency, and get the next patient by polling the highest urgency queue that is not empty.

Question 3. Using Maps. [22 marks]

Suppose you are writing a program for a web site that lets people vote for their favourite songs. To vote, users enter the name of their song. Your program uses a Map (stored in the votes field) to keep track of all the songs that have been entered and the number of votes for each song. The song names (Strings) are the keys of the map, and the number of votes for the song are the values.

```
private Map<String, Integer> votes = new HashMap<String, Integer>();
```

(a) [7 marks] Complete the following addVote method that will record another vote in the votes map. The parameter is the name of the song. Note that if this is the first vote for a song, the name of the song will not yet be in the votes map, and must be added.

```
public void addVote(String song){
    if (votes.containsKey(song)){
        votes.put(song, votes.get(song)+1);
    }
    else {
        votes.put(song, 1);
    }
//OR (more efficient — only two accesses )
    Integer count = votes.get(song);
    if (count == null) {
        votes.put(song, 1);
    }
    else {
        votes.put(song, count + 1);
}   }
```

(Question 3 continued on next page)

(b) [8 marks] Complete the following topSong method that will return the song with the highest number of votes.

```
    public String topSong(){
        int maxCount = -1;
        String tSong = null;
        for (String song : votes.keySet()){
            int count = votes.get(song);
            if (count > maxCount){
                maxCount = count;
                tSong = song;
            }
        }
        return tSong;
    }
//OR
    public String topSong2(){
        int maxCount = -1;
        String tSong = null;
        for (Map.Entry<String,Integer> entry : votes.entrySet()){
            if (entry.getValue() > maxCount){
                maxCount = entry.getValue();
                tSong = entry.getKey();
            }
        }
        return tSong;



    }
```

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

(Question 3 continued)

(c) [7 marks] Suppose your addVote method took 10 milliSeconds when votes had 1000 different songs and you were adding a new vote for a song that was already in votes. Estimate the time addVote would take to add a new vote for an existing song if votes contained 1,000,000 different songs for each of the following cases. Justify your answer and show any working.

(i) Estimated time given that the votes field contains a hashMap.

10 milliseconds. Searching in a hash table (as long as it is not full), is O(1) and therefore has approximately the same cost whatever the size of the table.

(ii) Estimated time if the votes field contained an arrayMap where the song–count pairs were stored in an unordered array.

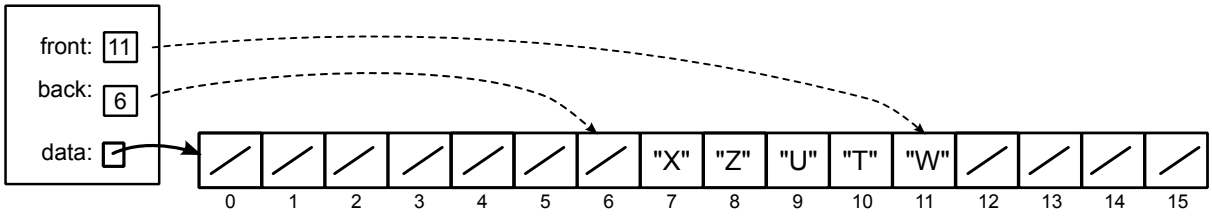
10 seconds (= 1000 × 10 milliseconds). It has to search through the array: the cost is O(n). Since 1,000,000 songs is 1000 times 1000 songs, the cost of searching will be about 1000 times greater than 10 ms.

(iii) Estimated time if the votes field contained a sortedArrayMap where the song–count pairs were stored in a sorted array, and the methods used binary search to find a song.

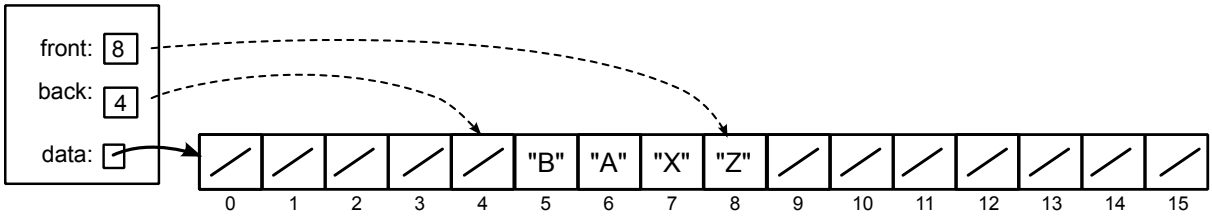
20 milliseconds. It has to do binary search on the array: the cost is O(log(n)). Searching 1000 songs required about 10 steps, searching 1,000,000 songs requires about 20 steps, therefore the time will be roughly double. The cost may be a little less since there is some fixed overhead.

Question 4. Implementing a Collection [15 marks]

This question concerns a YarraQueue class which implements a Queue collection using an array. Part of the YarraQueue class is shown below. YarraQueue objects have a data field to hold the array of values, a front field to hold the position of the value at the front of the queue, and a back field to hold the position one before the value at the back of the queue. The diagram shows an example of a queue with 5 values.



Values are added at the back of the queue and removed from the front of the queue. For example if "A" and then "B" are added to the queue above and three items were removed, the queue would now be:



When a queue is empty, both front and back will contain the value data.length-1. When a queue is not empty, back will be less than front.

```
public class YarraQueue <E> extends AbstractQueue <E> {

    private static final int INITIALCAPACITY = 16;
    private E[ ] data;
    private int front ;
    private int back;

    public YarraQueue(){
        data = (E[ ]) new Object[INITIALCAPACITY];
        back = data.length−1;                          // Queue is initially empty
        front = data.length−1;
    }
    public boolean isEmpty(){
       return (back == data.length−1);
    }
    public boolean offer(E item){
        if (item == null) throw new NullPointerException("null argument");
        if (back<0) ensureCapacity();
        data[back] = item;
        back−−;
        return true;
    }
```

(Question 4 continued on next page)

(Question 4 continued)

(a) [4 marks] Complete the following size method, which should return the number of items in the queue. The cost of your size method should not depend on the number of items in the queue.

```
public int size () {
    return front - back;



}
```

(b) [11 marks] Complete the following poll method, which should remove and return the value at the front of the queue. If the queue is empty, it should return null. If it removes the last value in the queue, front and back should be set to the appropriate value.

```
public E poll (){
    if (isEmpty()) return null;
    E ans = data[front];
    data[front] = null;
    front--;
    if (front<=back){
        back = data.length-1;
        front = data.length-1;
    }
    return ans;









}
```

Question 5. Binary Search. [15 marks]

Consider the following find method that uses binary search to find the correct location for a value in a sorted ArrayList of Strings. It contains a line of debugging code that will print out the values of low and high at the end of each iteration of the loop.

```
private int find(String item, ArrayList<String> valueList){
    int low = 0;
    int high = valueList.size();
    while (low < high){
        int mid = (low + high) / 2;
        if (item.compareTo(valueList.get(mid)) > 0)
            low = mid + 1;
        else
            high = mid;
        UI.println(low +" "+ high);   // Debugging Code
    }
    return low;
}
```

(a) [6 marks] Suppose the ArrayList names contains the the following values:

"ant"	"bee"	"cat"	"dab"	"eel"	"fox"	"gnu"	"hen"	"jay"	"kea"
0	1	2	3	4	5	6	7	8	9

If find were called with the arguments:
find("dog", names)
what will the debugging code print out, and what value will be returned?

low	high
0	5
3	5
3	4
4	4

Value returned: 4

(Question 5 continued on next page)

(Question 5 continued)

(b) [5 marks] Complete the following containsValue method that returns true if a string is in the given arrayList and false otherwise. It assumes the arrayList is sorted. Your answer should use the find method on the facing page.

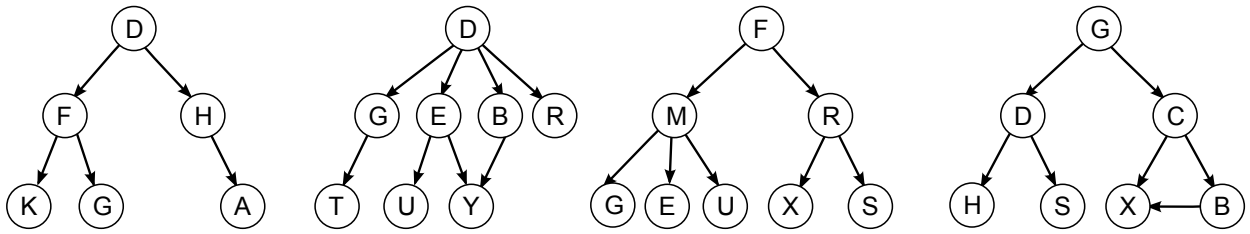
```
public boolean containsValue(String value, ArrayList<String> valueList){
    if (value == null) return false;
    int idx = find(value, valueList);
    if (idx >= valueList.size) return false;
    return value.equals(valueList.get(idx));
    // if check done other way round, have to check whether valueList.get(idx) is null
}
```

(c) [4 marks] Explain why the cost of the find method is always O(log(n)), if the ArrayList contains n values.

In every case, find will halve the size of the region it is looking in (low..high) each time round the loop. It is only possible to half *size* elements log(*size*) times.

Question 6. Trees [25 marks]

(a) [6 marks] For each of the following trees, state whether it is a binary tree, a ternary tree, a general tree, or not a tree, and give a brief justification of your answer.



(A) (B) (C) (D)

(A) binary because no nodes have more than two children. (Is also a ternary tree and a general tree.)

(B) Not a tree, because node X has two parents

(C) Ternary tree, because the non-leaf nodes have three children

(D) Not a tree, because some of the nodes have links to their siblings

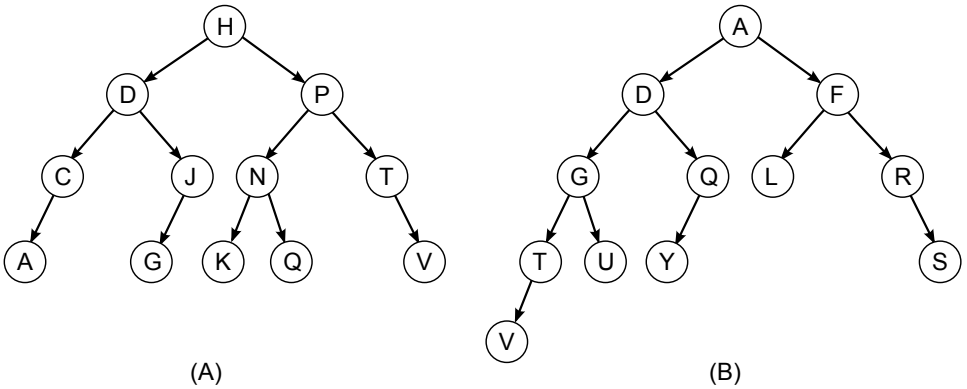
(Question 6 continued on next page)

(Question 6 continued)

(b) [8 marks] Each of the following binary trees may be

- a binary search tree (BST), a partially ordered tree(POT), or neither of these.
- complete or not complete
- balanced or not balanced

Circle the correct option for each tree and give a brief justification of your answers. Assume that the items are to be ordered alphabetically.



(A) ~~BST / POT~~ / Neither
Justification: Not BST because J is in the left subtree of H but is greater than H. Similarly Q is greater than P. Not POT because children are not all greater than their parents.

~~Complete~~ / Not complete
Justification: Although all the leaves are in the bottom two rows, the leaves on the bottom row are not all to the left.

Balanced / ~~Not balanced~~
Justification: because all the paths to the leaves are the same length.

(B) ~~BST~~ / POT / ~~Neither~~
Justification: because every node comes before its children

~~Complete~~ / Not complete
Justification: because one of the leaves (L) is not in the bottom two rows

~~Balanced~~ / Not balanced
Justification: the paths to the leaves are of different lengths (3, 4, or 5). Note that the tree is balanced under the AVL definition of balanced because at every node, the height of the left subtree is within 1 of the height of the right subtree.

(Question 6 continued on next page)

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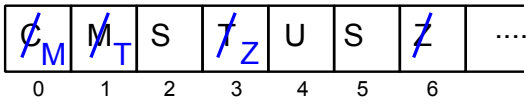
Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

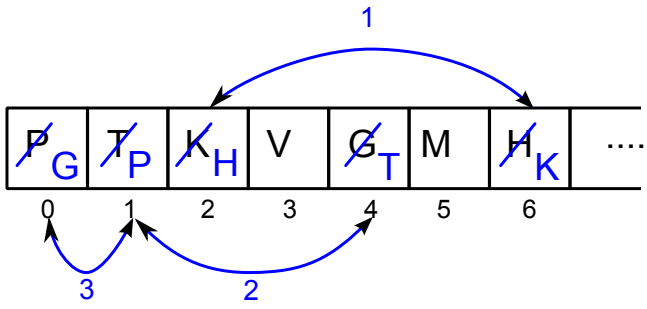
(Question 6 continued)

A heap is a complete, partially ordered tree stored in an array.

(c) [5 marks] Show the state of the following heap after the highest ranked value (C) is removed from the heap.
Hint: draw the tree.



(d) [6 marks] Convert the following array of values into a heap. Use labeled arrows to show the sequence of swaps that are required.

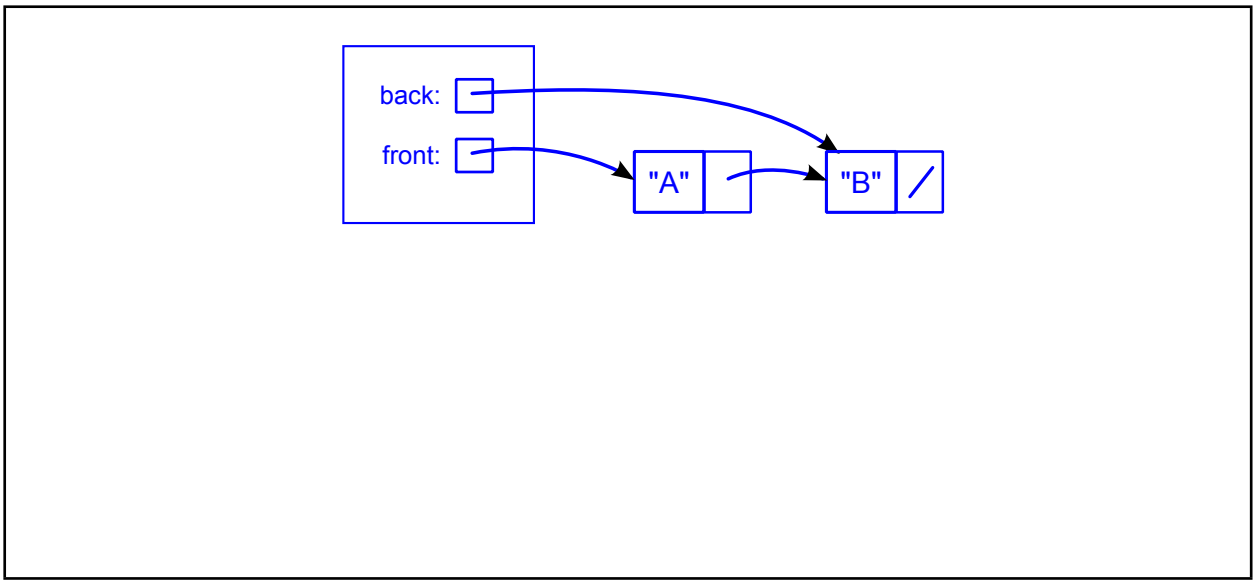


Question 7. Linked Lists. [25 marks]

Suppose you are writing a LinkedQueue class that implements the Queue interface, using a linked list to hold the values. Your implementation uses the following LinkedNode class.

```
public class LinkedNode<E>{
    private E value;
    private LinkedNode<E> next;
    public LinkedNode(E v, LinkedNode n){
        value = v;
        next = n;
    }
    public E getValue(){return value;}
    public LinkedNode getNext(){return next;}
    public void setNext(LinkedNode n){next = n;}
}
```

(a) [5 marks] Draw a diagram of a LinkedQueue that contains the two values "A" and "B". Show the LinkedQueue object, its fields, and the LinkedNode objects.



(b) [2 marks] Give definitions of the fields of the LinkedQueue class:

```
public class LinkedQueue <E> extends AbstractQueue<E>{

    private LinkedNode<E> front;
    private LinkedNode<E> back;
```

(Question 7 continued on next page)

(Question 7 continued)

(c) [3 marks] Complete the following isEmpty method for the LinkedQueue class:

```
public boolean isEmpty(){
    return (back==null);




}
```

(d) [8 marks] Complete the following poll method of the LinkedQueue class. Note, the cost of your poll method should be O(1).

```
public E poll(){
    if (this.isEmpty()) return null;
    E ans = front.getValue();
    front = front.getNext();
    if (front==null){
        back == null;
    }
    return ans;









}
```

(Question 7 continued on next page)

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

(Question 7 continued)

(e) [7 marks] Complete the following offer method of the LinkedQueue class. Note, the cost of your offer method should be O(1).

```
public boolean offer(E value){
        if  (value == null)  return false;
        if  (this.isEmpty()){
              back = new LinkedNode(value, null);
              front  = back;




        } else {
              back.setNext(new LinkedNode(value, null));
              back = back.getNext();









        }
        return true;
}
```

Question 8. Tree Traversals [13 marks]

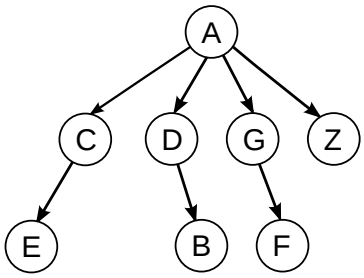
Consider the following traversalDFS method that does a depth first traversal of a general tree, printing out values in the nodes.

```
public void traversalDFS(GenTreeNode root){
    if (root==null) return;
    Stack<GenTreeNode> stack = new Stack<GenTreeNode>();
    stack.push(root);
    while (!stack.isEmpty()){
        GenTreeNode node = stack.pop();
        UI.print(node.getValue()+"  ");
        for(GenTreeNode child : node.getChildren()){
            stack.push(child);
        }
    }
}
```

(a) [2 marks] Does traversalDFS do a pre-order or a post-order traversal of a tree? Justify your answer.

Pre-order, because it prints the node before it processes its children.

(b) [4 marks] List the output of the traversalDFS method if it were called on the following tree. Assume that the children of a node are shown in order from left to right.



A Z G F D B C E

note that it puts all the children on to the stack in order so that the rightmost child is at the top of the stack.

(Question 8 continued on next page)

(Question 8 continued)

(c) [7 marks] Modify the traversal method above so that it does a breadth first traversal of a general tree instead of a depth first traversal. It should print the above tree in the order A C D G Z E B F.

```
public void traversalBFS(GenTreeNode root){
    if (root==null) return;
    Queue<GenTreeNode> queue = new ArrayQueue<GenTreeNode>();
    queue.offer(root);
    while (!queue.isEmpty()){
        GenTreeNode node = queue.poll();
        UI.print(node.getValue()+"  ");
        for(GenTreeNode child : node.getChildren()){
            queue.offer(child);
        }
    }




}
```

Question 9. Hashing [18 marks]

(a) [6 marks] The following table shows a mapping between characters and the hash values generated for those characters. Insert the characters into the array below, starting at the top row and working your way down. Use linear probing to resolve collisions.

A	⇒	4
B	⇒	2
C	⇒	4
D	⇒	5
E	⇒	3
F	⇒	3

0	1	2	3	4	5	6	7
-	-	B	E	A	C	D	F

(b) [6 marks] Consider the following (bad) hash function. Explain what makes it bad and what effect it would have on a hash table.

```
public int hash(String data) {
    int value = 0;
    foreach (char ch: data) {
        value += ch + Math.random();
    }
    return value;
}
```

The use of Math.random means that you will get a different hash value each time you call it, so you will never be able to find an item again, after you have put it in the hash table.

(Question 9 continued on next page)

(Question 9 continued)

(c) [6 marks]

The following remove method is for a HashSet that uses linear probing to resolve collisions, and has a good hash method. Explain why this method might result in data (other than the item to be removed) being lost.

You may assume that the data field points at a valid array.

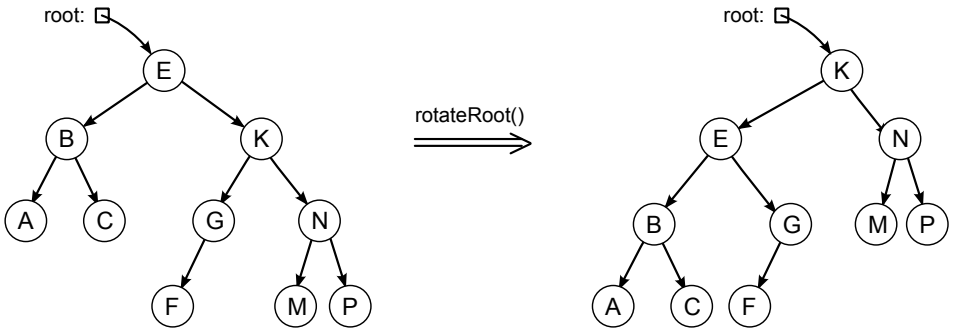
```
private String[ ] data;
private int hash(String item) { ... }

public boolean remove(String item) {
    int startIndex = hash(item) % data.length;
    int index = startIndex;
    while (true) {
        if (data[index] == null) return false;
        if (data[index].equals(item)) {
            data[index] = null;
            return true;
        }
        index = (index + 1) % data.length;
        if (index == startIndex) return false;
    }
}
```

Because it replaces the item it removes by null, it will lose any values that hashed to that space or spaces on its left, but due to linear probing were placed in spaces to its right. A later search for any such items will be stopped at the null, and prevent the search from finding the items.

Question 10. Debugging Tree Code [14 marks]

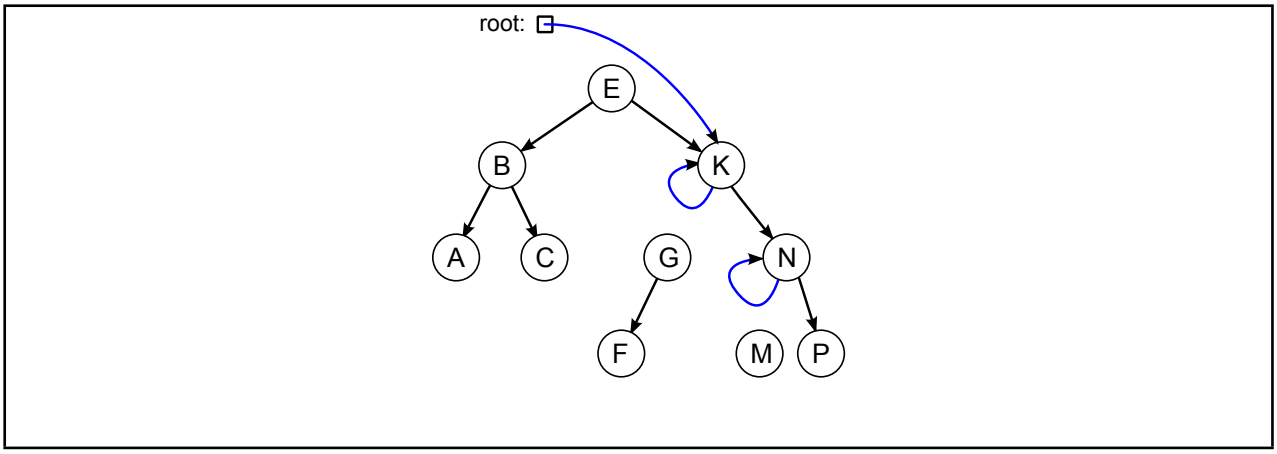
The following rotateRoot method is intended to "rotate" the root of a Binary Search Tree and its right child without losing the BST ordering property. For example, it should change the tree on the left into the tree on the right. The current root is moved to the left, its right child becomes the root, and one of its grandchildren is also moved to the left.

However, rotateRoot has errors.




```
    private BSTNode root;   // contains the root node of a Binary Seach Tree
     :
    public void rotateRoot(){
        root = root.right;
        root.left = root;
        root.right.left = root.left.right;
    }

    private class BSTNode {
        public String value;
        public BSTNode left;
        public BSTNode right;
        public BSTNode(String v, BSTNode lf, BSTNode rt){value = v; left=lf; right=rt;}
    }
}
```

(a) [6 marks] Draw the tree that will actually result if rotateRoot is called on the tree on the left above.



(Question 10 continued on next page)

(Question 10 continued)

(b) [8 marks] Fix the errors in rotateRoot

```
public void rotateRoot(){      // assumes that root is a field of type BSTNode
    if (root==null || root.right== null) return;
    BSTNode oldRoot = root;
    BSTNode oldRightLeft = root.right.left ;

    root = oldRoot.right;
    root.left = oldRoot;
    root.left.right = oldRightLeft;









}
```

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Specify the question number for work that you do want marked.

Appendix (may be removed)

Brief (and simplified) specifications of some relevant interfaces and classes.

```
interface Collection<E>
    public boolean isEmpty()
    public int size()
    public boolean add(E item)
    public boolean contains(Object item)
    public boolean remove(Object element)
    public Iterator<E> iterator()

interface List<E> extends Collection<E>
    // Implementations: ArrayList, LinkedList
    public E get(int index)
    public E set(int index, E element)
    public void add(int index, E element)
    public E remove(int index)
    // plus methods inherited from Collection

interface Set extends Collection<E>
    // Implementations: ArraySet, HashSet, TreeSet
    // methods inherited from Collection

interface Queue<E> extends Collection<E>
    // Implementations: ArrayQueue, LinkedList, PriorityQueue
    public E peek ()                    // returns null if queue is empty
    public E poll ()                    // returns null if queue is empty
    public boolean offer (E element)    // returns false if fails to add
    // plus methods inherited from Collection

class Stack<E> implements Collection<E>
    public E peek ()                    // returns null if stack is empty
    public E pop ()                     // returns null if stack is empty
    public E push (E element)           // returns element being pushed
    // plus methods inherited from Collection

interface Map<K, V>
    // Implementations: HashMap, TreeMap, ArrayMap
    public V get(K key)                 // returns null if no such key
    public V put(K key, V value)        // returns old value, or null
    public V remove(K key)              // returns old value, or null
    public boolean containsKey(K key)
    public Set<K> keySet()              // returns a Set of all the keys

class Collections
    public static void sort(List<E>)
    public static void sort(List<E>, Comparator<E>)
    public static void shuffle(List<E>, Comparator<E>)
```

```
class Arrays
    public static <E> void sort(E[ ] ar, Comparator<E> comp);

class Random
   public int nextInt( int n);   // return a random integer between 0 and n−1
   public double nextDouble(); // return a random double between 0.0 and 1.0

interface Iterator <E>
   public boolean hasNext();
   public E next();
   public void remove();

interface Iterable <E>              // Can use in the "for each" loop
   public Iterator <E> iterator();

interface Comparable<E>             // Can compare this to another E
   public int compareTo(E o);       // −ve if this less than o; +ve if greater than o;

interface Comparator<E>             // Can use this to compare two E's
   public int compare(E o1, E o2); // −ve if o1 less than o2; +ve if greater than o2
```