



EXAMINATIONS — 2012

MID YEAR

COMP103
Introduction to
Data Structures and Algorithms

Time Allowed: 3 Hours

- Instructions:**
1. Attempt **all** of the questions.
 2. *Read each question carefully before attempting it.* (Suggestion: You do not have to answer the questions in the order shown. Do the questions you find easiest first.)
 3. This examination will be marked out of **180** marks, so allocate approximately one minute per mark.
 4. Write your answers in the boxes in this test paper and hand in all sheets.
 5. Non-electronic translation dictionaries are permitted.
 6. Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination.
 7. Documentation on some relevant Java classes, interfaces, and exceptions can be found at the end of the paper.

Questions	Marks
1. Collections	[30]
2. Sorting	[30]
3. Recursion and Linked Lists	[25]
4. Trees	[35]
5. Heaps	[20]
6. Hashing	[30]
7. Final Hard Question	[10]

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Specify the question number for work that you do want marked.

Question 1. Collections

[30 marks]

(a) [3 marks] You are writing a program that needs to store a collection of values with duplicates. Java does not have a class called **Bag** in its `java.util` package. Which `java.util` class would you chose to use instead? Justify your answer.

(b) [3 marks] What is the best general purpose implementation of a set in `java.util`? Justify your answer.

(c) [3 marks] State *two* ways in which *merge sort* is better than *quick sort*.

(d) [3 marks] State *two* ways in which *quick sort* is better than *merge sort*.

(Question 1 continued on next page)

(Question 1 continued)

Assume that the class `MyMap` below is intended to implement a map. It uses an `ArrayList` of `Entry` objects each containing a key and value pair.

(e) [8 marks] Initialise the `data` field as appropriate and implement a `put` method that puts a mapping from key to value into the map and replaces the existing mapping if key is already in this map.

```
import java.util.*;
public class MyMap<K, V> {
    class Entry<K, V> {
        public K key;
        public V value;
        public Entry(K k, V v) { this.key = k; this.value = v; }
    }
    ArrayList<Entry<K, V>> data =

    public void put(K key, V value) {

    }
}
```

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Cross out rough working that you do not want marked.
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Question 2. Sorting

[30 marks]

(a) [10 marks] Consider the following array of letters. Draw a diagram of merge sort working on this array. Show how it divides and combines the parts resulting in the sorted version.

L	Y	G	P	Y	O	K	A	B	E	Q	X	Z	N	M	G
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



(Question 2 continued on next page)

(Question 2 continued)

Suppose quick sort is called on the following array. Assume the pivot is chosen to be the element in the first position of the subarray being processed (*not the median of first, middle, and last value as was done in the lectures!*). The first step is shown, you need to show the second and the third steps, circling the pivots involved as shown.

L	Y	G	P	Y	O	K	A	B	E	Q	X	Z	N	M	G
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

After the first split (note the circle around the pivot):

G	K	A	B	E	G	L	Y	P	Y	O	Q	X	Z	N	M
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

(b) [5 marks] After the next split of both left and right parts (circle pivot used in left part and circle pivot used in right part):

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

(c) [5 marks] After the next split of each of the four parts above (circle the four pivots):

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

You *do not* need to show any further steps.

(Question 2 continued on next page)

(Question 2 continued)

(d) [10 marks] Here is our array for one last time. Show how it changes as you sort it using selection sort. For every step clearly identify the sorted and unsorted parts. *STOP when half of the array is sorted.*

L	Y	G	P	Y	O	K	A	B	E	Q	X	Z	N	M	G
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 3. Recursion and Linked Lists

[25 marks]

Use recursion to implement a `LinkedList` class representing a single node of a linked list. The fields and the constructor are given to you below. You will need to complete three additional recursive methods: `size`, `add`, `remove`.

```
public class LinkedListNode<E> {  
    public E item;  
    public LinkedListNode<E> next;  
  
    public LinkedListNode(E item, LinkedListNode<E> next) {  
        this.item = item; this.next = next;  
    }  
}
```

(a) [3 marks] Implement a recursive method that returns the size of a linked list when called on the head node of the list.

```
public int size() {
```

```
}
```

(Question 3 continued on next page)

(Question 3 continued)

(b) [4 marks] Implement a recursive method that adds an item to the linked list when called on the head node of a linked list. If the item is already present in the list, it should return false and not add the item. If the item is not present, it should add the item and return true.

```
public boolean add(E item) {
```

```
}
```

(c) [3 marks] Implement a recursive method that removes an item from a linked list when called on the head node of a linked list. The method should return false if and only if the item is not present. *You can assume that the item is NOT in the first node of the list.*

```
public boolean remove(E item) {
```

```
}
```

```
}
```

(Question 3 continued on next page)

(Question 3 continued)

(d) [5 marks] Give a better version of the `remove` method in the `LinkedList` class that also handles the case where the item is in the first node. Assume that the list *contains at least 2 items*.



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(Question 3 continued)

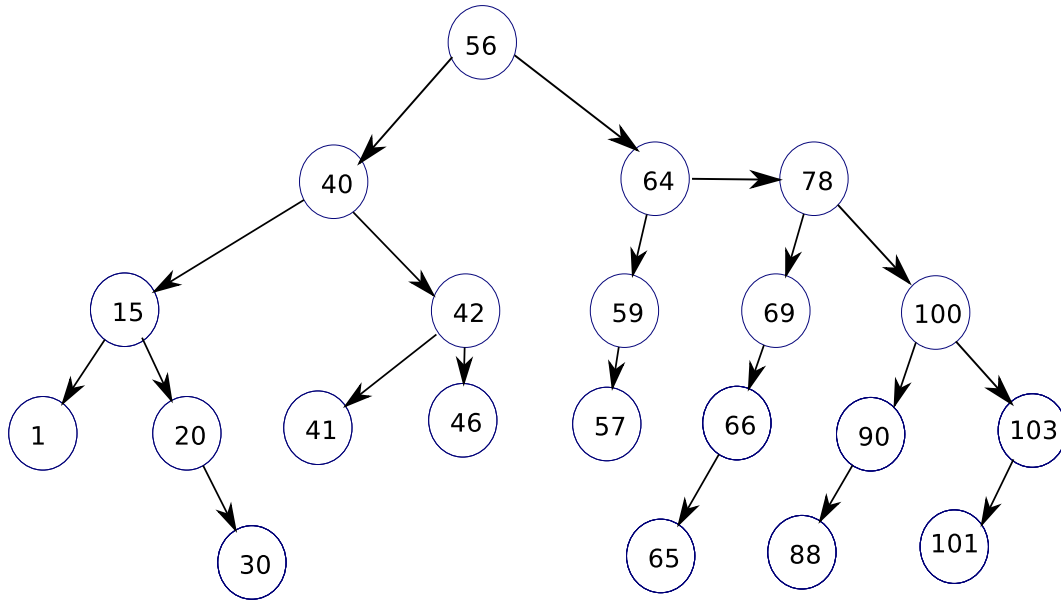
(e) [5 marks] Would you be able to write a version of the `remove` method in the `LinkedList` class that can remove the first item of a list with only one item in it? If yes, then outline the code and explain why it would work, if not, then explain and justify why not.

(f) [5 marks] Explain how you would use the `LinkedList` class to implement a *queue*. Would you need to adjust the behaviour of the `LinkedList` class to make it work for a general queue? Justify your answer.

Question 4. Trees

[35 marks]

(a) [10 marks] Consider the following Binary Search Tree that is used to implement a Set. Show what happens as you add the following 5 items (*using the algorithm discussed in lectures*) in the following order: 0, 55, 68, 69, 102.



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(Question 4 continued)

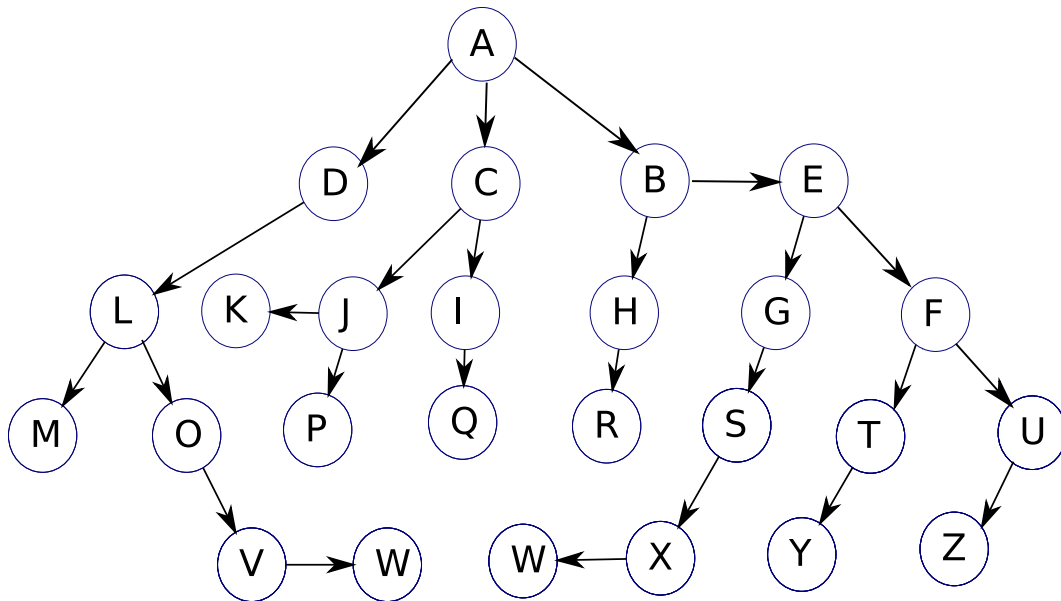
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(Question 4 continued)

Consider the following general tree:



(b) [5 marks] List the nodes in the *breadth first* order of traversal (always processing children of a node left to right).

(c) [5 marks] List the nodes in the *pre-order depth first* order of traversal (always processing children of a node left to right).

(Question 4 continued)

You are given a class `GeneralTreeNode` that implements a node of a general tree:

```
import java.util.*;
```

```
public class GeneralTreeNode<E> {  
    public E item;  
    public Set<GeneralTreeNode<E>> children = new HashSet<GeneralTreeNode<E>>();
```

(d) [5 marks] Write a recursive method on a `GeneralTreeNode` that returns the maximum *depth* of the tree rooted at the current node. A tree of only one node has depth 1.

```
public int depth() {
```

```
}
```

(Question 4 continued on next page)

(Question 4 continued)

(e) [10 marks] Write a recursive method `levelWidth` that returns the width of a given level inside a tree. Assume the root is at level 1.

```
public int levelWidth(int level) {
```

```
}
```

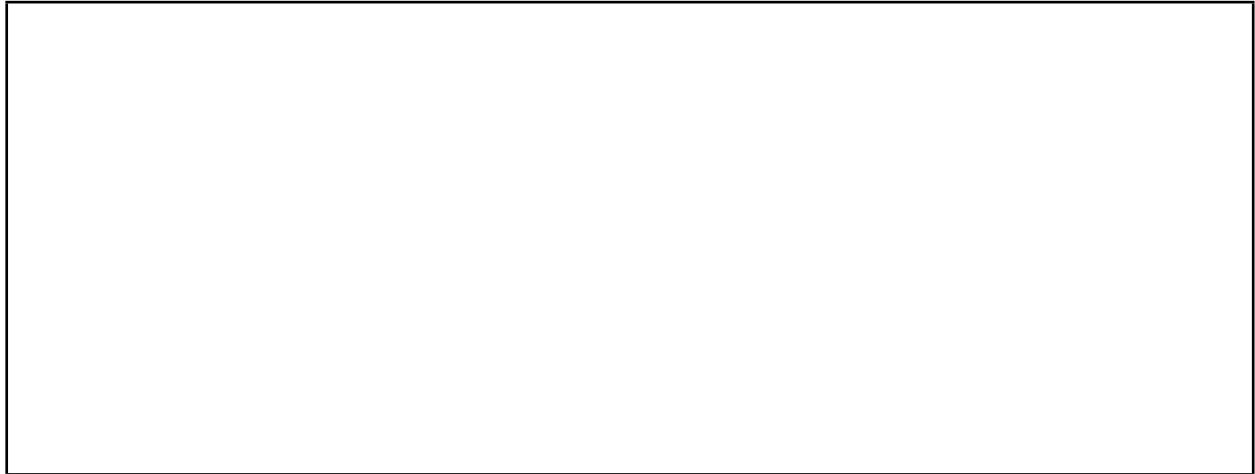
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Question 5. Heaps

[20 marks]

(a) [5 marks] A heap is a *complete partially ordered binary tree stored in an array*. Why is it important for this tree to be complete? Justify your answer.

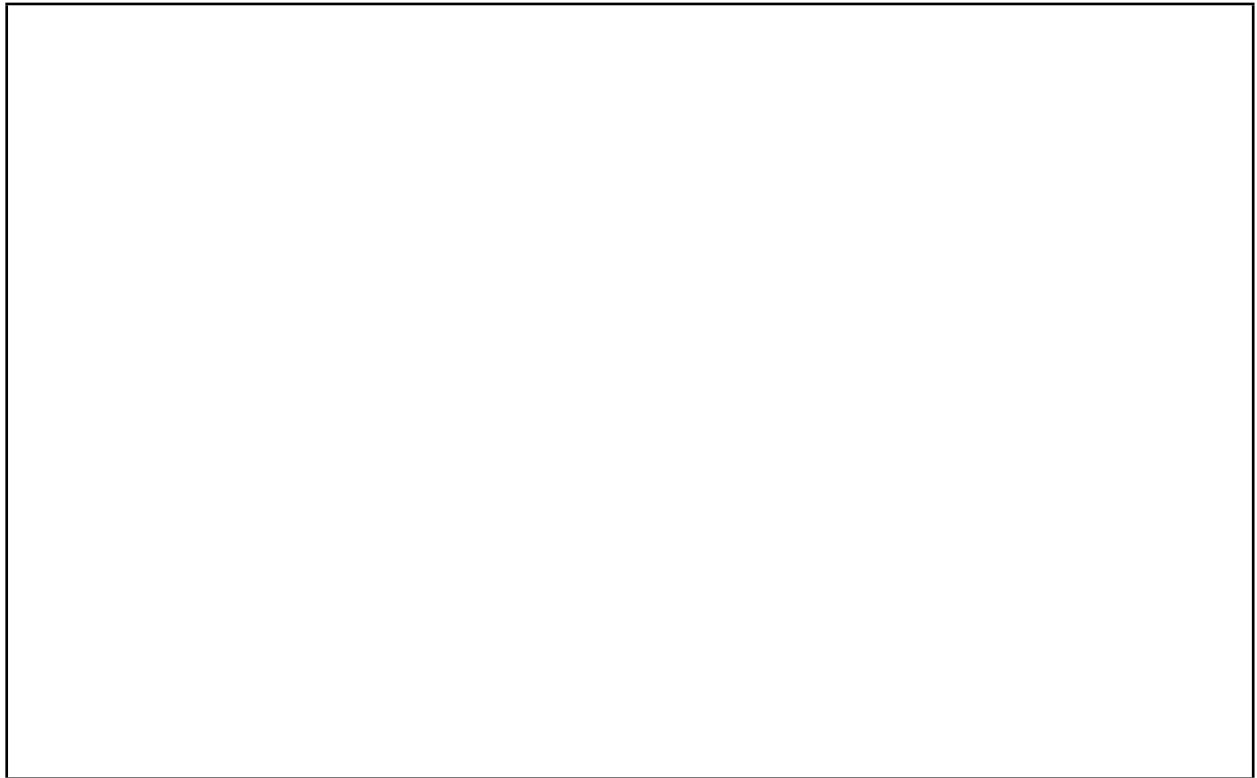


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(Question 5 continued)

(b) [4 marks] Draw the following heap as a tree.

Z	R	Q	H	I	L	M	A	B	D	C				
---	---	---	---	---	---	---	---	---	---	---	--	--	--	--



(c) [3 marks] Given the following heap (assume that the letters coming later in the alphabet have to be *higher* in the partially ordered tree), perform the following operation.

Z	R	Q	H	I	L	M	A	B	D	C				
---	---	---	---	---	---	---	---	---	---	---	--	--	--	--

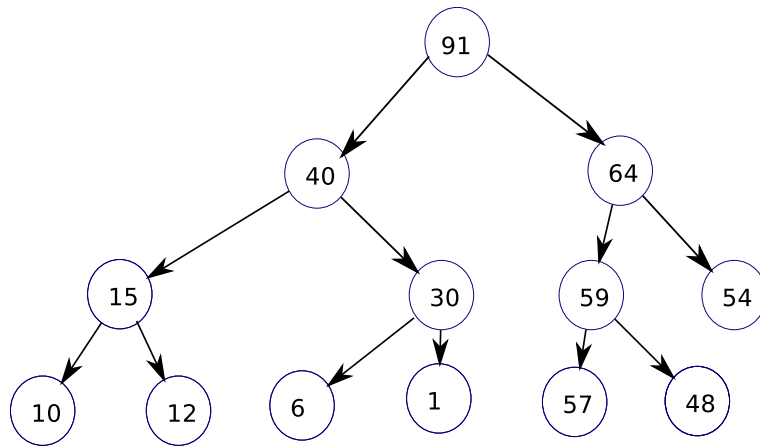
Add P:

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

(Question 5 continued on next page)

(Question 5 continued)

Consider the following heap drawn as a tree:



(d) [3 marks] Show how the heap drawn as a tree above would be represented in the array.

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

(e) [5 marks] Draw the resulting tree after 91 is removed:

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Question 6. Hashing

[30 marks]

Consider the following class:

```
public class Computer {  
    char[] tag = new char[6];  
    String manufacturer;  
    int HDDSize;  
    int RAMSize;  
    boolean has3DgraphicsCard;  
    String modelNumber;  
}
```

(a) [4 marks] Given an example of a *bad* hash function for this class and justify why it is not going to be useful for hashing.

(b) [6 marks] Given an example of a *good* hash function for this class (that does not use a memory address!) and justify why it is going to be useful for hashing.

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(Question 6 continued)

(c) [10 marks] Implement the `add` method for the `HashSet` class below. Assume that you will never make this `HashSet` full and use *linear probing* for collision resolution. Feel free to utilise the default `hashCode` function as defined in `java.lang.Object`. *You should not assume that any helper functions such as `contains` are present.*

```
public class HashSet<E> {
    public final int INIT_CAPACITY = 7;
    E[] data = (E[]) new Object[INIT_CAPACITY];

    public boolean add(E element) {

    }
}
}
```

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(Question 6 continued)**(d)** [10 marks] Implement an Iterator for your HashSet above.

```
public class HashSet<E> implements Iterable<E> {  
    public final int INIT_CAPACITY = 7;  
    E[] data = (E[]) new Object[INIT_CAPACITY];  
  
    public java.util.Iterator<E> iterator() { return new HashSetIterator(); }  
  
    class HashSetIterator implements java.util.Iterator<E> {  
  
        public HashSetIterator() {  
  
        }  
  
        public boolean hasNext() {  
  
        }  
  
        public E next() {  
  
        }  
  
        public void remove() {  
            throw new UnsupportedOperationException();  
        }  
    }  
}
```

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Cross out rough working that you do not want marked.
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Question 7. Final Hard Question

[10 marks]

(a) [10 marks] Implement the following `split` method for your `LinkedList` class from Question 3 that operates *recursively* and is initially called on the first node of the linked list.

- For a single node list, the value returned is null.
- For a list with 2 or more nodes that has an *even* number of nodes, the value returned is the head of the second half of the list and the original list is shortened to only contain the first half.
- For a list with 2 or more nodes that has an *odd* number of nodes, the first half list should be one node longer than the second half list.

For example, if you call this `split` method on list: `{1,2,3,4,5}` then `{4,5}` will be returned and the original list will become `{1,2,3}`. If you call it on `{1,2,3,4}` then `{3,4}` will be returned and the original list will become `{1,2}`.

Note that Java's `/` operator returns the truncated integer value as in: $5/2 = 2$ and $4/2 = 2$ and $7/2 = 3$.

```
public LinkedList<E> split() {
```

```
}
```

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(Question 7 continued)

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Cross out rough working that you do not want marked.
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Appendix (may be removed)

Brief (and simplified) specifications of some relevant interfaces and classes.

```
public class UI extends java.lang.Object
```

```
    public static void clearGraphics();
```

```
    public static void setColor(java.awt.Color col);
```

```
    public static void drawOval(double x, double y, double width, double height);
```

```
    public static void fillOval (double x, double y, double width, double height);
```

```
public class Scanner
```

```
    public boolean hasNext(); // there is more to read
```

```
    public String next(); // return the next token (word)
```

```
    public String nextLine(); // return the next line
```

```
    public int nextInt (); // return the next integer
```

```
public class Random
```

```
    public int nextInt( int n); // return a random integer between 0 and n-1
```

```
    public double nextDouble(); // return a random double between 0.0 and 1.0
```

```
public interface Iterator <E>
```

```
    public boolean hasNext();
```

```
    public E next();
```

```
    public void remove();
```

```
public interface Iterable <E>
```

```
    public Iterator <E> iterator();
```

```
// Can use in the "for each" loop
```

```
public interface Comparable<E>
```

```
    public int compareTo(E o);
```

```
// Can compare this to another E
```

```
public interface Comparator<E>
```

```
    public int compare(E o1, E o2);
```

```
// Can use this to compare two E's
```

```
public interface Collection<E>
    public boolean isEmpty();
    public int size ();
    public boolean add(E element);
    public Iterator <E> iterator();
```

```
public interface List<E> extends Collection<E>
    // Implementations: ArrayList
    public E get(int index);
    public void set(int index, E element);
    public void add(E element);
    public void add(int index, E element);
    public void remove(int index);
    public void remove(Object element);
```

```
public interface Set extends Collection<E>
    // Implementations: HashSet, SortedSet, TreeSet
    public boolean contains(Object element);
    public boolean add(E element);
    public boolean remove(Object element);
```

```
public interface Queue<E> extends Collection<E>
    // Implementations: PriorityQueue, LinkedList
    public E peek (); // returns null if queue is empty
    public E poll (); // returns null if queue is empty
    public boolean offer (E element);
```

```
public class Stack<E> implements Collection<E>
    public E peek (); // returns null if stack is empty
    public E pop (); // returns null if stack is empty
    public E push (E element);
```

```
public interface Map<K, V>
    // Implementations: HashMap, TreeMap, LinkedHashMap
    public V get(K key); // returns null if no such key
    public void put(K key, V value);
    public void remove(K key);
    public Set<Map.Entry<K, V>> entrySet();
```