Student ID: .....

TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

VICTORIA
UNIVERSITY OF WELLINGTON

EXAMINATIONS – 2013

TRIMESTER 1

COMP 103
INTRODUCTION TO DATA STRUCTURES AND ALGORITHMS

Time Allowed: THREE HOURS

Instructions: Closed Book.

Attempt ALL Questions.

Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found.

The exam will be marked out of 180 marks.

Documentation on some relevant Java classes, interfaces, and exceptions can be found at the end of the paper. You may tear that page off if it helps.

There are spare pages for your working and your answers in this exam, but you may ask for additional paper if you need it.

Only silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted in this examination.

Non-electronic foreign language dictionaries are permitted.

Questions	**Marks**
1. Collections	[12]
2. Using collections	[28]
3. Implementing a Collection	[17]
4. Binary Search	[12]
5. Recursion and Sorting	[21]
6. Linked Lists	[15]
7. Stacks, Queues	[7]
8. Trees, Tree Traversals	[28]
9. Partially Ordered Trees, and Heaps	[16]
10. Hashing	[17]
11. Various	[7]

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 1. Collections [12 marks]

(a) [2 marks] A List differs from a Set in two main ways. What are these differences?

List can have duplicates, unlike a Set. A List has an ordering, unlike a Set.

(b) [2 marks] In Java, what happens to an object when it is no longer referenced by any other objects?

The memory it occupies is made available to other processes - "garbage collection".

(c) [2 marks] How many levels are there in a complete binary tree having 32 nodes?

6

(d) [2 marks] TreeSort is based on what search algorithm?

Binary Search

(e) [2 marks] Why does it not make sense to do an in-order traversal of a Ternary Tree?

In-order tranversal visits the left, then the node itself, then the right: it's specific to binary trees, whereas ternary trees have up to 3 children per node.

(f) [2 marks] If you want to sort a collection on a device with a small amount of memory, which of the fast sorting algorithms we studied (MergeSort, QuickSort, TreeSort and HeapSort) would you *not* employ, and why not?

MergeSort and TreeSort, because they require a second data array - QuickSort, HeapSort are "in place" and therefore require much less memory.

Question 2. Using collections [28 marks]

This question is about a program called SongsOrganiser, which manages information about a playlist of Songs. The following code for a Song class just stores information:

```
public class Song {
    private int year;
    private String artist, title;

    // constructor
    public Song(String artist, String title, int yr) {
        this.year = yr;
        this.artist = artist;
        this.title = title;
    }
    // some 'get' methods to provide information.
    public String getArtist() { return artist; }
    public String getTitle() { return title; }
    public Integer getYear() { return year; }
}
```

A real SongsOrganiser program would read in data from a large file. The demo version shown here creates Song objects explicitly, and stores them in a List of Songs:

```
public class SongsOrganiser {
    // constructor
    public SongsOrganiser() {
        List <Song> playlist = new ArrayList <Song> ();

        playlist.add(new Song("Phoenix Foundation", "Hitchcock",2005));
        playlist.add(new Song("Phoenix Foundation", "Hitchcock",2005));
        playlist.add(new Song("Phoenix Foundation", "Buffalo",2010));
        playlist.add(new Song("Dresden Dolls", "Shores of California",2007));
        playlist.add(new Song("Tiny Ruins", "Priest with balloons",2011));
        playlist.add(new Song("SJD","Lena",2012));
        playlist.add(new Song("Azelia Banks", "212",2011));
        playlist.add(new Song("SJD", "Beautiful haze",2007));
        playlist.add(new Song("SJD", "Beautiful haze",2007));
        playlist.add(new Song("Phoenix Foundation", "Buffalo",2010));

        // re−order the list
        reorderPlaylist(playlist);

        // remove duplicates
        playlist = removeDuplicates(playlist);
    }
    :
}
```

(Question 2 continued)

The method reorderPlaylist() in SongsOrganiser takes the list and reorders it by using a comparator. The first question below asks you to write the comparator, the second asks for reorderPlaylist(), and the third asks for removeDuplicates().

(a) [10 marks] In the box below complete the code for SongComparator. This should compare Songs based on their year, artist, and title, *in that order*. That is, if two Songs differ in their year it should order them on that basis, but if the years are the same, it should compare them by their artist field using the usual alphabetical ordering. And if they still agree, it should compare the titles.

Note: in Java, strings are Comparable: a String can call the "compareTo(String another)" method. This compares two strings lexicographically (ie. the usual alphabetical ordering).

```
/** Comparator that will order Songs based on their year and title. */
private class SongComparator implements Comparator <Song> {

    public int compare (Song song1, Song song2) {
        // There are lots of ways to do this. Here is one.
        if (song1.getYear() − song2.getYear() != 0)
            return (song1.getYear() − song2.getYear());
        if (song1.getArtist().compareTo(song2.getArtist() !=0)
            return song1.getArtist().compareTo(song2.getArtist());
        return song1.getTitle().compareTo(song2.getTitle());

    }

}
```

(b) [6 marks] In the box below, complete the reorderPlaylist method, which should first create a comparator and then use that to sort the items in the list.

Note: you can do this question even if you did not complete the previous one, as it simply *uses* the Comparator.

Hint: use the Collections class given in the Appendix.

```
public void reorderPlaylist(List<Song> playem) {
    Collections.sort(playem, new SongComparator());
}
```

.

(Question 2 continued)

(c) [12 marks] The playlist in the preceding code may have duplications in it, which we would like to remove while preserving the order.

Write a method removeDuplicates for the SongsOrganiser class which takes a List of Song objects, and returns a List that is in the *same order*, but has any duplicates removed.

Note: there are several ways to answer this.

HINT: it might help to write out the necessary steps as "pseudocode" first.

```
public List<Song> removeDuplicates(List<Song> playem) {

    List <Song> newlist = new ArrayList <Song> ();
    for (Song s1 : playem)
        {
            boolean present = false;
            for (Song s2 : newlist)
                if (s1.getYear() − s2.getYear() == 0)
                    if (s1.getArtist().compareTo(s2.getArtist()) == 0)
                        if (s1.getTitle().compareTo(s2.getTitle()) == 0)
                            present = true;
            if (present == false) newlist.add(s1);
        }
    return newlist;
}
```

.

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 3. Implementing a collection [17 marks]

Suppose you wanted to implement a Stack using an array. The "top" of the Stack is going to correspond to the last of the items in the array (*i.e.* the one with the highest index).

Here is partial code for such an ArrayStack. The constructor and the pop method are given. This question asks you to code the four other methods given at the end.

```
public class ArrayStack <E> {
    private static int INITIALCAPACITY=3;
    private int count=0;
    private E[] data;

    public ArrayStack() { data =(E[]) new Object[INITIALCAPACITY]; }

    public E pop(){
        if (count == 0) return null;
        E saved = data[count−1];
        data[count−1] = null;
        count−−;
        return saved;
    }

    public boolean isEmpty(){ ... }
    public E peek(){ ... }
    public E push(E value){ ... }
    private void ensureCapacity(){ ... }
}
```

(a) [2 marks] In the box below, write code to implement the isEmpty() method.

```
    public boolean isEmpty(){
        return (count == 0);
    }
```

.

(b) [3 marks] In the box below, write code to implement the peek() method.

```
    public E peek(){
        if (count == 0) return null;
        return data[count−1];
    }
```

.

(Question 3 continued)

(c) [6 marks] In the box below, write code to implement the push() method. Your code should include a call to ensureCapacity() at some point, in case the array is full.

```
public E push(E value){
    if (value == null)
        return value;
    else {
        ensureCapacity();
        data[count] = value;
        count++;
        return value;
    }
}
```

.

(d) [6 marks] Complete the ensureCapacity() method below, which doubles the size of the data array being used, if the array becomes full.

```
private void ensureCapacity(){
    int L = data.length;
    if (count == L) {
        E[] newdata =(E[]) new Object[2*L];
        for (int i=0; i<L; i++)
            newdata[i] = data[i];
        data = newdata;
    }
    return;
}
```

.

Question 4. Binary Search. [12 marks]

(a) [8 marks] The find() method of Binary Search applied to a sorted ArrayList returns the index where the item would be (regardless of whether it is actually present).

In the box below, complete the find() method, which is passed a sorted ArrayList of Strings. *Note*: Java strings are Comparable.

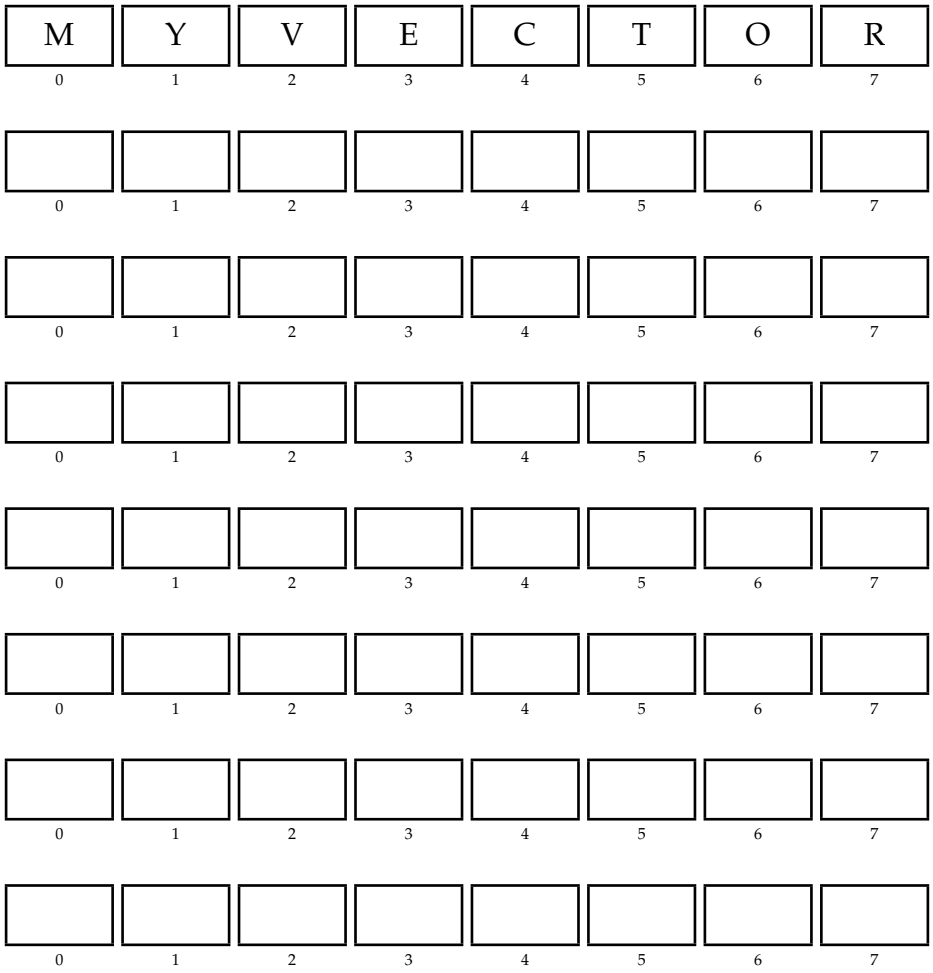
```
private int find(String item, ArrayList<String> valueList){
    int low = 0;
    int high = valueList.size();



    while (low < high){
        int mid = (low + high) / 2;
        if (item.compareTo(valueList.get(mid)) > 0)
            low = mid + 1;
        else
            high = mid;

    }
    return low;

}
```

(b) [4 marks] In words, explain why the cost of the find() method is always $\mathcal{O}(\log n)$, if the ArrayList contains n values.

In every case, find() will halve the size of the region it is looking in (low..high) each time round the loop. It is only possible to halve n elements log(n) times.

Question 5. Recursion and Sorting [21 marks]

(a) [5 marks] Starting with the following array of letters, complete the steps that the InsertionSort algorithm follows to yield an alphabetically sorted array.
(*Note:* you may not need all the rows that are given).



(b) [4 marks] In words, explain why the InsertionSort algorithm has cost $\mathcal{O}(n^2)$ on lists that begin randomly ordered, but has cost $\mathcal{O}(n)$ on lists that begin in an order that is already nearly sorted.

Cost on randomly ordered Lists is $\mathcal{O}(n^2)$, because...
InsertionSort repeatedly takes next item and finds where to put it in the List up to that point. The outer loop goes around n times, and each time runs an inner loop going backwards to figure out where the new item goes: this is linear in n, hence $\mathcal{O}(n^2)$.

Cost on nearly-ordered Lists becomes $\mathcal{O}(n)$, because...
The inner loop now only has one comparison to do (not $n/2$ or so), hence $\mathcal{O}(n)$.

(Question 5 continued)

(c) [4 marks] Write a method thirds() that uses recursion to keep dividing a number by 3 using integer division, as long as the number remains positive. Your method should print out the results as it goes. For example, calling

```
thirds(300);
```

should print this:

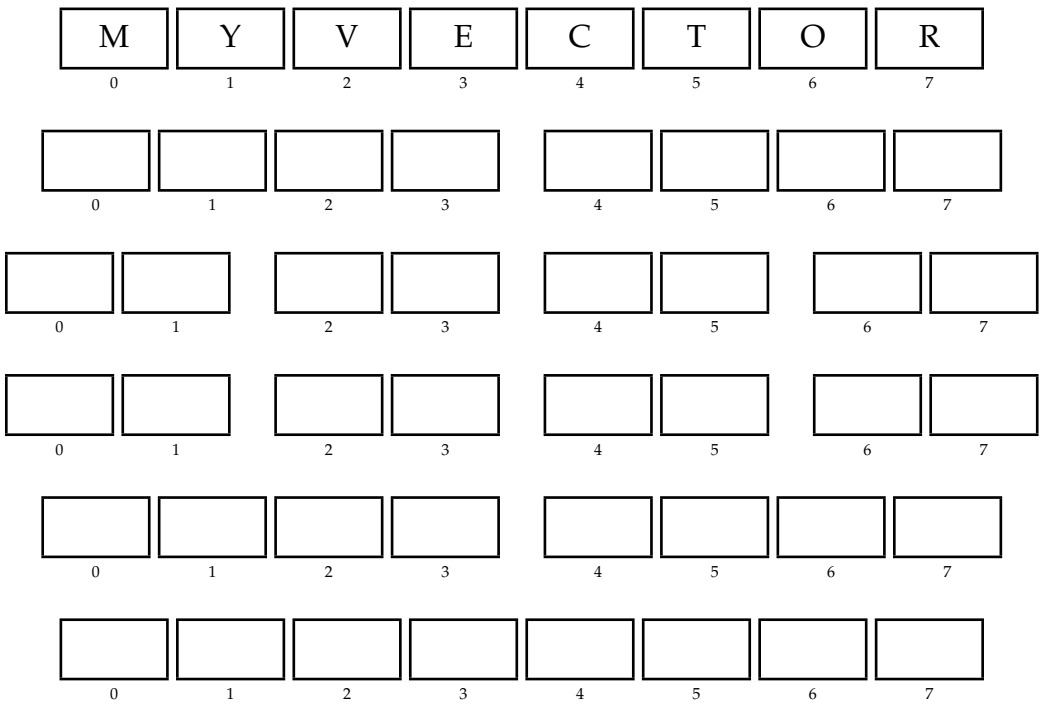
```
300    100    33    11    3    1
```

```
public void thirds(int n) {
    if (n>0) {
        UI.print(n + "  ");
        thirds(n/3);
    }
    else return;
}
```

(d) [3 marks] What is the "big-$\mathcal{O}$" cost of the thirds(n) method, in terms of its argument n?

If it were halves instead of thirds, the recursion will go $\log_2(n)+1$ deep (the number of halvings...), so the big-oh cost would be $\mathcal{O}(\log n)$. Changing to thirds only alters a multiplier on this.

(e) [5 marks] Suppose MergeSort is run on the following array of letters. Complete the steps to yield an array that is sorted alphabetically.



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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 6. Linked Lists. [15 marks]

(a) [15 marks] Suppose you are writing a program to help keep track of a sports team. Each player is represented with an instance of class Player, and the list of players on the team is represented with a linked list. Sometimes, one player may be replaced by another player. Write the code that finds a player in the list of players, and replaces that linked list node with a new node that contains the new player.

Your implementation should use the following LinkedNode class:

```
public class LinkedNode<E>{
    private E value;
    private LinkedNode<E> next;
    public LinkedNode(E v, LinkedNode n){
        value = v;
        next = n;
    }
    public E getValue(){return value;}
    public LinkedNode getNext(){return next;}
    public void setNext(LinkedNode n){next = n;}
}
```

You will not need to make use of any methods of the Player class except its equals() method. In the method header below, the player to replace is passed in the parameter oldPlayer, and the new player is in the parameter newPlayer.

```
LinkedNode<Player>
    listReplace(LinkedNode<Player> listhead, Player oldPlayer, Player newPlayer) {
(medium/hard)
// recursive
    if (head == null) return null;
    if (head.getValue().equals(oldplayer)) { // replace first
      LinkedNode<String> newhead = new LinkedNode<String>(newplayer, head.getNext());
      return newhead;
    }
    else {
      head.setNext( listReplace(head.getNext(), oldplayer, newplayer ) );
      return head;
    }

// alternately, iterative
    if (head == null) return null;
    if (head.getValue().equals(oldplayer)) { // replace first
      LinkedNode<String> newhead = new LinkedNode<String>(newplayer, head.getNext());
      return newhead;
    }
    while( head.getNext() != null ) {
      if (head.getNext().equals(oldplayer)) {
        head.setNext( newplayer, head.getNext().getNext() );
        return;
      }
      head = head.getNext()
   } //while
}
```

Question 7. Stacks, Queues. [7 marks]

(a) [5 marks] Consider the following code that uses a stack:

```
Stack<Integer> s = new Stack<Integer>();
s.push(15);
s.push(8);
Integer i1 = s.pop();
System.out.println(i1);
s.push(25);
Integer i2 = s.pop();
System.out.println(i2);
Integer i3 = s.pop();
System.out.println(i3);
```

What will be printed when this code is run?

(easy question)
8 25 15

(b) [2 marks] Which of these (if any) are appropriate uses for a queue:
(i) Keeping a sorted list of students, and their grades
(ii) Keeping the next-to-visit nodes for a depth-first traversal of a tree
(iii) Keeping the next-to-visit nodes for a breadth-first traversal of a tree

Write the number(s) in the answer box below, or write "none".

(easy/medium question) (c)

Question 8. Trees, Tree Traversals [28 marks]

For this question, we define an empty tree to have depth 0, a tree with only a root to have depth 1, a tree with a root and only one leaf to have depth 2, etc.

(a) [2 marks] What is the depth of a full binary tree that contains three nodes?

(easy)
2

(b) [3 marks] Now consider a full binary tree whose depth is *one greater* than in the previous question. How many nodes are in this tree?

(easy)
7

(c) [4 marks] How many leaves are there in the bottom layer of a tree with depth 7?

(medium)
$2^6, = 64.$

(d) [2 marks] What is the depth of a balanced binary tree with n nodes?

(easy)
$\log_2 n$. Answering $\log n$ is also fine.

(e) [2 marks] What is the maximum depth of an *unbalanced* binary tree with n nodes?

(easy)
n

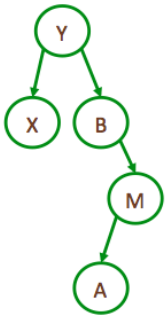
(f) [1 mark] Consider a full, balanced binary tree with 32 layers (depth = 32). In describing the number of nodes in this tree, which of these would be the most appropriate comparision:

(i) several thousand
(ii) $\log_2(32)$
(iii) the number of squares in half of a chess board
(iv) the number of people in the world (about 7 billion)

(advanced)
(d). 2^{32} = 4.x billion, world population = 6.x billion. The other 3 answers can be ruled out fairly easily.

(g) [4 marks] What will the following method print if called on the root of the tree shown below?

```
void treeprint(BinaryTreeNode node) {
    System.out.println(node.value);
    if (node.leftChild != null) treeprint(node.leftChild);
    if (node.rightChild != null) treeprint(node.rightChild);
}
```



(easy/medium)
Y X B M A

(h) [10 marks] Suppose you have a binary tree that contains integers. Here is a partial description of a node in that tree:

```
public class TreeNode {
  public int getValue()
  public void setValue(int value)
  public TreeNode getLeftChild()
  public TreeNode getRightChild()
}
```

Now assume that every leaf has its integer value set, but the values at the other nodes need to be set.

Every node in this tree has either two children, or none. There are no nodes with only one child.

Write a method for the TreeNode class that, at each node, computes the difference between the value of its left child and the value of its right child (that is, left - right). It should then set the value of the node to this difference.

medium/hard

```
// todo sanity check this
int getDifference(TreeNode n)
{
  if ((getLeftChild()==null) && (getRightChild()==null))
    return getValue();
  else {
    int value = getDifference(getLeftChild()) − getDifference(getRightChild());
    setValue(value);
    return value;
  }
}
```

.

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
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Question 9. Partially Ordered Trees, and Heaps [16 marks]

(a) [2 marks] Under best-case assumptions, which is the fastest data structure for implementing contains() (i.e. set membership: "Is a given item in the set?") :

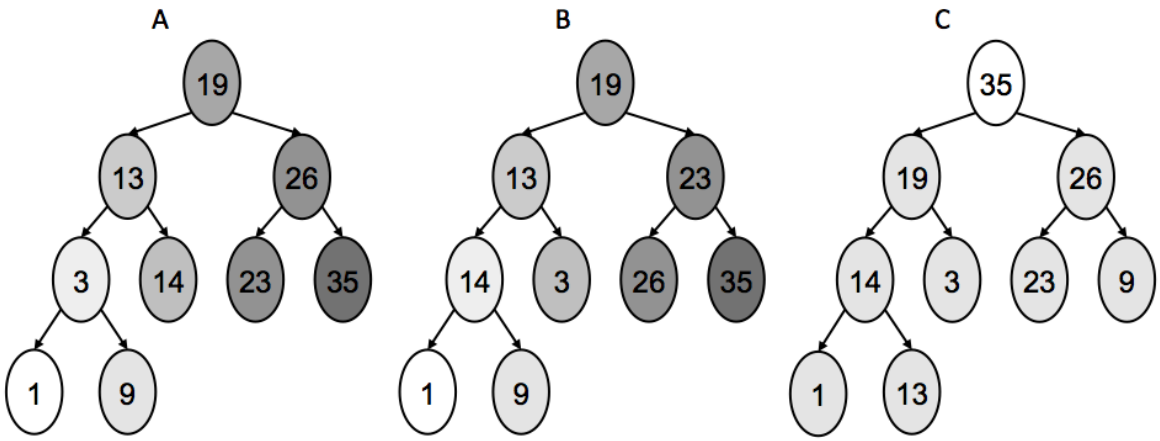
- a binary search tree,
- a heap, or
- a hash table.

Justify your answer.

(easy) hash table

Justification O(1) lookup

(b) [4 marks] Of the trees labeled A,B,C below, one is a binary search tree (BST), one is a partially ordered tree (POT), and one is neither. For each of A,B,C, write what type of tree it is.



(easy)
A BST

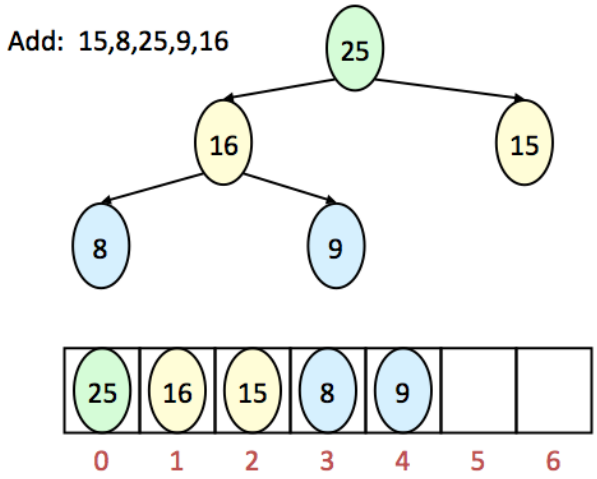
B neither

C POT

(c) [10 marks] Show the resulting heap when the following values are added to an empty heap, in this order: 15, 8, 25, 9, 16 .

This is a "max" heap, with the largest value at the root (not the smallest).

Draw both the partially-ordered tree representation, and the array representation of the heap.



Question 10. Hashing [17 marks]

(a) [4 marks] Briefly describe what "Collision" means in hashing:

(medium)
Different objects can have the same hash code, and thus map to the same location in a hash table.

(b) [2 marks] Hash tables can be used to implement maps, sets, bags, and other data structures. Consider a hash table of size 100, where each entry in the array is a linked list storing items whose hash code is the index of that array cell (i.e., this hash table implements a set).

After entering 10 such items in the table, what is the complexity ('big $\mathcal{O}$' cost) of checking whether a given item is already in the table? Assume that the hash function is well suited for the data that is entered in the table.

(medium)
O(1)

(c) [2 marks] Now assume that millions of different items have been entered in the table, but the table itself is still of size 100. What is the complexity of looking up whether a given item is in the table? Briefly explain your answer.

(hard)
Justification: O(n), or perhaps O(n/100) if we consider 1/100 to be significant.

(d) [2 marks] Now consider using a hash table to implement a bag rather than a set. If you start with an empty bag, and add the same item many times, what is the complexity of looking up whether some item (possibly different) is in the table? Briefly explain your answer.

(hard)
Justification: O(1). Most items map to an empty cell, so O(1). If you're looking for the item that has been entered multiple times, you'll find it immediately. The bad case is an item that collides with the multiply-entered item, but this case is rare.

(e) [3 marks] Suppose we want to make a hash table containing the citizens of New Zealand. We are looking for a good hash key. One possibility is to use the town or city where each person lives, mapped to an integer, as the hash key. That is, we make a list of towns in New Zealand, sorted by size in decreasing order, and use the index of the town as the hash key. People living in Auckland (population 1.4 million) would have key 0, Wellington (population 395,000) would have key 1, etc.

Is this a good hash key? Briefly explain your answer.

(medium)
no
Justification: quite uneven distribution

(f) [4 marks] You are given a Person class that includes information on the person's date of birth. That information is stored as a set of three integers, one for the year, one for the month, and one for the day of birth. Here is a partial description of the Person class:

```
public class Person implements Comparable<Person> {
  public Person(String name, int day, int month, int year)
  public String getName()
  public int getYearOfBirth()
  public int getMonthOfBirth()
  public int getDayOfBirth()
  public int hashCode()
 ...
}
```

The hashCode method is defined as follows:

```
  int hashCode() {
      return getYearOfBirth();
  }
```

Now we'll add the following people to a hash table:

Name	*Date of birth*
Hamish Skywalker	1/2/95
Jon Smith	28/07/93
Amy Dilbert	5/12/93
Bill Smith	11/11/90
Bertha Duckworth	28/9/72

How many people will collide in the hash table? Can you give their names?

(medium/hard)
at least two: Jon/Amy

give 2 marks for saying Jon/Amy,
2 more marks for saying "at least".

Question 11. Various [7 marks]

(a) [2 marks] Describe what is meant by a "Stable" sort.

(medium)

items that compare as equal should not change place during the sort.

(b) [5 marks] As background for this question, recall that SelectionSort has $\mathcal{O}(n^2)$ complexity. An intuitive explanation of this is that there are n items, and each item requires $\mathcal{O}(n)$ complexity to select from among the remaining unsorted items. Thus, n items each with complexity $\mathcal{O}(n)$ gives $n \times \mathcal{O}(n) = \mathcal{O}(n^2)$.

Consider one of the algorithms you studied that has $\mathcal{O}(n \log_2 n)$ complexity (HeapSort might be an easy example, but you can choose another $\mathcal{O}(n \log_2 n)$ algorithm).

For the algorithm you choose, give a brief explanation (similar to the explanation for SelectionSort given above) of where the n comes from, and where the $\log_2 n$ comes from.

(advanced)
E.g. mergesort, each step touches all n items, and there are log n steps.

****

Appendix (may be removed)

Brief (and simplified) specifications of some relevant interfaces and classes.

```
interface Collection<E>
    public boolean isEmpty()
    public int size()
    public boolean add(E item)
    public boolean contains(Object item)
    public boolean remove(Object element)
    public Iterator<E> iterator()

interface List<E> extends Collection<E>
    // Implementations: ArrayList, LinkedList
    public E get(int index)
    public E set(int index, E element)
    public void add(int index, E element)
    public E remove(int index)
    // plus methods inherited from Collection

interface Set extends Collection<E>
    // Implementations: ArraySet, HashSet, TreeSet
    // methods inherited from Collection

interface Queue<E> extends Collection<E>
    // Implementations: ArrayQueue, LinkedList, PriorityQueue
    public E peek () // returns null if queue is empty
    public E poll () // returns null if queue is empty
    public boolean offer (E element) // returns false if fails to add
    // plus methods inherited from Collection

class Stack<E> implements Collection<E>
    public E peek () // returns null if stack is empty
    public E pop () // returns null if stack is empty
    public E push (E element) // returns element being pushed
    // plus methods inherited from Collection

interface Map<K, V>
    // Implementations: HashMap, TreeMap, ArrayMap
    public V get(K key) // returns null if no such key
    public V put(K key, V value) // returns old value, or null
    public V remove(K key) // returns old value, or null
    public boolean containsKey(K key)
    public Set<K> keySet() // returns a Set of all the keys

class Collections
    public static void sort( List<E>)
    public static void sort( List<E>, Comparator<E>)
    public static void shuffle ( List<E>, Comparator<E>)
```

```
class Arrays
    public static <E> void sort(E[ ] ar, Comparator<E> comp);

class Random
   public int nextInt( int n);   // return a random integer between 0 and n−1
   public double nextDouble(); // return a random double between 0.0 and 1.0

interface Iterator <E>
   public boolean hasNext();
   public E next();
   public void remove();

interface Iterable <E>              // Can use in the "for each" loop
   public Iterator <E> iterator();

interface Comparable<E>             // Can compare this to another E
   public int compareTo(E o);       // −ve if this less than o; +ve if greater than o;

interface Comparator<E>             // Can use this to compare two E's
   public int compare(E o1, E o2); // −ve if o1 less than o2; +ve if greater than o2
```