

Name:

ID Number:

COMP103: Test

10 Sept, 2003.

Instructions

- Time: **2 hours**.
- Answer **all** the questions.
- There are 120 marks in total.
- Write your answers in the boxes in this test paper and hand in all sheets.
- Every box with a heavy outline requires an answer.
- If you think some question is unclear, ask for clarification.
- We expect the later questions to be more difficult than the earlier ones.

Questions

Marks

1. Collection Types	[12]	<input type="text"/>
2. Hash Tables	[10]	<input type="text"/>
3. Asymptotic or “Big O” analysis	[18]	<input type="text"/>
4. Programming with Collections	[21]	<input type="text"/>
5. Cost of Programs	[13]	<input type="text"/>
6. Timing Performance of Bags	[16]	<input type="text"/>
7. Linked Lists	[16]	<input type="text"/>
8. Union for Sets	[14]	<input type="text"/>

There is documentation on the `jds` types at the end of the exam paper.

Total:

Possibly useful formulas:

- $1 + 2 + 3 + 4 + \dots + k = \frac{k(k+1)}{2}$
- $1 + 2 + 4 + 8 + \dots + 2^k = 2^{k+1} - 1$
- $a + (a + b) + (a + 2b) + \dots + (a + kb) = \frac{(2a+kb)(k+1)}{2}$
- $a + as + as^2 + as^3 + \dots + as^k = \frac{as^{k+1} - a}{s - 1}$

Question 1. Collection Types

[12 marks]

Match each of the jds types in the left column to a description in the right hand column by drawing lines from the types to the descriptions. Note: some descriptions do not match any type.

Types	Descriptions
Set	Collection of values Order of values is not meaningful Duplicate values not allowed.
	Collection of key – value pairs Order of values is the order in which their keys were added. Duplicate values allowed
Indexed	Collection of values Order of values is the order in which the values were added. Duplicate values not allowed
Map	Collection of key – value pairs Order of values and keys is not meaningful Only one value may be associated with any key
	Collection of values Order of values is the “natural” sorting order of the values Duplicate values not allowed
Bag	Collection of values Order of values is not meaningful Duplicate values allowed
	Collection of values Order of values is the order in which the values were added. Duplicate values allowed
	Collection of key – value pairs Order of values is “natural” sorting order of the keys. Duplicate keys allowed, but values must be unique.

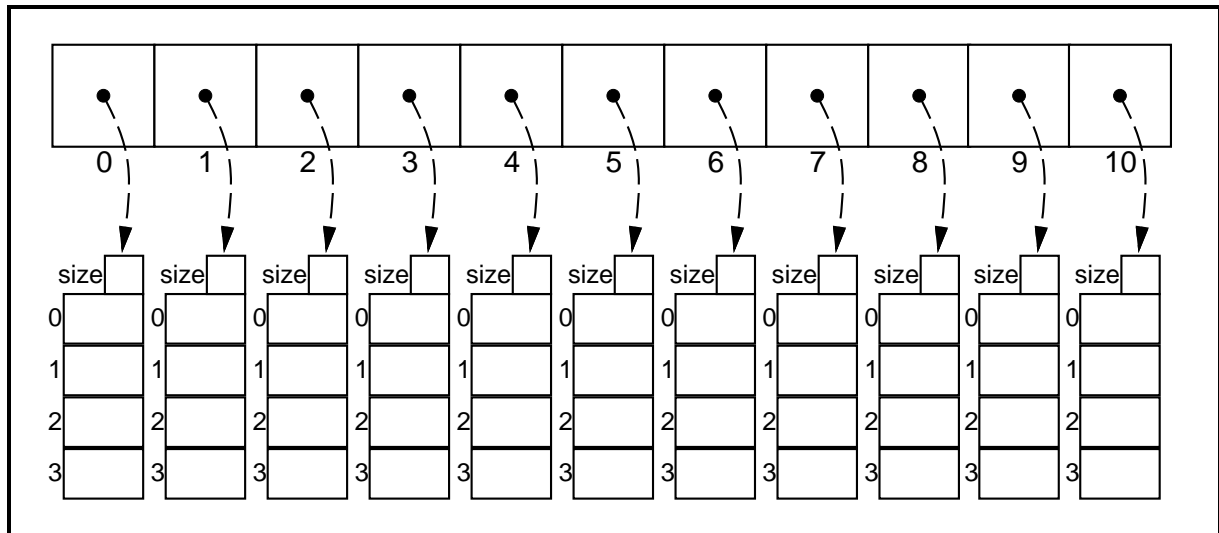
Question 2. Hash Tables

[10 marks]

(a) [5 marks] The diagram below shows a Set implemented using a BucketHashSet with 11 buckets. Each bucket is implemented as an ArraySet.

Show (on the diagram) the contents of the Set when the following seven items are added to the set in the order “Hen”, “Owl”, “Cat”, etc.

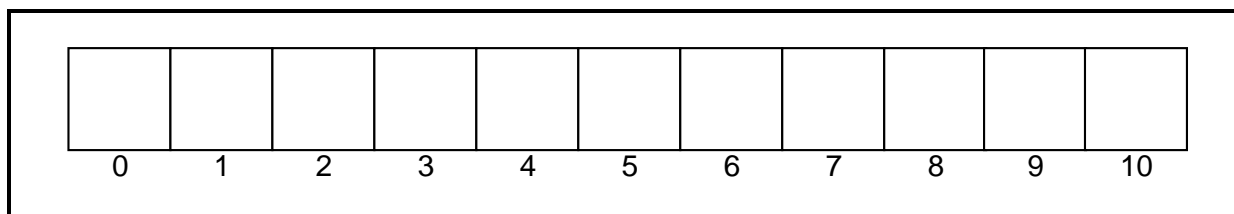
item:	Hen	Owl	Cat	Cow	Bee	Emu	Yak
hash value:	9	1	10	4	9	4	0



(b) [5 marks] The diagram below shows a Set implemented using an OpenHashSet that uses linear probing.

Show (on the diagram) the contents of the Set when the following seven items are added to the Set in the order “Hen”, “Owl”, “Cat”, etc.

item:	Hen	Owl	Cat	Cow	Bee	Emu	Yak
hash value:	9	1	10	4	9	4	0



Question 3. Asymptotic or “Big O” analysis

[18 marks]

(a) [4 marks] What is the average case asymptotic (“Big O”) costs of searching for an item in each of the following implementations of Set? Assume that the size of the Set is n .

- **ArraySet**
(unordered, array)

$O(\quad)$

- **SortedArraySet**
(ordered, array)

$O(\quad)$

- **BucketHashSet**
(Hashtable with k buckets where each bucket is an ArrayBag)

$O(\quad)$

- **OpenHashSet**
(Hashtable with open addressing, linear probing, and guaranteed less than 90% full).

$O(\quad)$

(b) [5 marks] What are the average case asymptotic costs (“Big O”) of the following sorting algorithms?

- **Bubble Sort**

$O(\quad)$

- **Insertion Sort**

$O(\quad)$

- **Merge Sort**

$O(\quad)$

- **Quicksort**

$O(\quad)$

- **Selection Sort**

$O(\quad)$

(Question 3 continued on next page)

(Question 3 continued)

(c) [3 marks] Suppose a program uses an algorithm with an average case asymptotic cost of $O(n)$. When the program is run on a case where $n = 1,000$, the measured running time is 4.5 seconds. When $n = 2,000$, the measured running time is 9.0 seconds. Give a reasonable estimate of the running time of the program on a case where $n = 4,000$. Justify your answer.

Estimated running time:

Justification:

(d) [3 marks] Suppose a program uses an algorithm with an average case asymptotic cost of $O(n^2)$. When the program is run on a case where $n = 1,000,000$, the measured running time is 3.5 minutes. Give a reasonable estimate of the running time of the program on a case where $n = 4,000,000$. Justify your answer.

Estimated running time:

Justification:

(e) [3 marks] Suppose a program uses an algorithm with an average case asymptotic cost of $O(\log(n))$. When the program is run on a case where $n = 1,000$, the measured running time is 0.7 seconds. When $n = 2,000$, the measured running time is 0.9 seconds. Give a reasonable estimate of the running time of the program on a case where $n = 8,000$. Justify your answer.

Estimated running time:

Justification:

Question 5. Cost of programs

[13 marks]

In each of the following, express your answer as a function of n . Do NOT use Big-O notation.

(a) [3 marks] Consider the following program fragment:

```
for (int i = 0; i < n; i++)
    System.out.println(i*i);
```

How many lines will be printed out by this piece of code?

(b) [3 marks] Consider the following program fragment:

```
for (int i = 1; i <= n; i++)
    for (int j = i; j <= n; j++)
        System.out.println(i + j);
```

How many lines will be printed out by this piece of code?

(c) [4 marks] Consider the following program fragment, assuming that **data** is an array of n Strings.

```
for (int i = 1; i < data.length; i++)
    if (data[i].compareTo(data[i-1]) < 0)
        System.out.println(data[i]+ " is out of order");
```

In the worst case, how many lines will be printed out by this piece of code?

In the best case, how many lines will be printed out by this piece of code?

(d) [3 marks] Consider the following program fragment, assuming that **data** is an array of Strings of length n , and that n is a power of 2.

```
int i = data.length;
while (i > 0){
    System.out.println(data[i-1]);
    i = i/2;
}
```

How many lines will be printed out by this piece of code?

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Question 6. Timing Performance of Bags

[16 marks]

The tables below give the results of a timing test on four different implementations of Bag (all run on the same computer):

- **ArrayBag**: an array of items, not kept in order
- **SortedArrayBag**: an array of items, kept in sorted order
- **BucketHashBag**: a hash table of 1000 buckets, where each bucket was a **SortedArrayBag**
- **OpenHashBag**: a hash table using open addressing, guaranteed to never be more than 80% full.

For each implementation of Bag, the test measured the average time (in microseconds) it took to search for an item (both for items in the Bag and for items not in the Bag) and the average time it took to add a new item to the bag when the bag contained n items. For each implementation, the test was run with four different values of n .

For each table, say which implementation of Bag it is, give the asymptotic (Big-O) cost of searching and adding in that implementation, and a brief justification of why the table must be from that implementation of Bag.

(a) [4 marks] Table 1:

n	μS per Search	μS per Add
5000	0.261	0.23
10000	0.371	0.23
20000	0.521	0.23
40000	0.631	0.31

Implementation:

Big-O costs:

Justification:

(b) [4 marks] Table 2:

n	μS per Search	μS per Add
5000	1.8	41
10000	2.5	100
20000	3.6	481
40000	4.8	1738

Implementation:

Costs:

Justification:

(Question 6 continued on next page)

(Question 6 continued)

(c) [4 marks] Table 3:

n	μS per Search	μS per Add
5000	0.41	0.51
10000	0.71	0.81
20000	1.06	1.07
40000	1.73	1.51

Implementation:

Costs:

Justification:

(d) [4 marks] Table 4:

n	μS per Search	μS per Add
5000	133	0.1
10000	363	0.1
20000	1159	0.1
40000	2520	0.1

Implementation:

Costs:

Justification:

Question 7. Linked Lists

[16 marks]

This question concerns the `SingleList` class that implements a collection using a linked list data structure to store the items. Like the `Indexed` type, the order of items in a `SingleList` is significant.

The `SingleList` class uses a singly linked list with no tail pointer. Part of the implementation is given below, along with a diagram of a `SingleList` object containing three items.

```
public class SingleList implements Collection {
    private Node data;
    :
    private class Node{
        public Object value;
        public Node next;
        public Node(Object val, Node n){
            value = val;
            next = n;
        }
    }
}
```

The diagram illustrates a singly linked list. A box labeled 'data' contains a pointer to the first node. The first node is a circle containing a box labeled 'Dog' and a pointer to the second node. The second node is a circle containing a box labeled 'Kea' and a pointer to the third node. The third node is a circle containing a box labeled 'Cat' and a null pointer (represented by a square with a diagonal line).

You are to complete several of the method definitions for `SingleList` class.

(a) [4 marks] Complete the following definition of the `removeFirst` method that removes the first item of the collection. It throws a `NoSuchElementException` exception if there are no items in the collection.

```
public void removeFirst(){
}
}
```

(Question 7 continued on next page)

(Question 7 continued)

(b) [4 marks] Complete the following definition of the `elementAt` method that returns the i 'th value in the list (counting from 0). It throws a `NoSuchElementException` if i is greater than or equal to the number of values in the list.

```
public Object elementAt (int i) {
```

```
}
```

(c) [4 marks] Complete the following definition of the `addLast` method that adds an item at the end of the collection. Note that the first element in the collection is at position 0.

```
public void addLast (Object val) {
```

```
}
```

(Question 7 continued on next page)

(Question 7 continued)

(d) [4 marks] Complete the following definition of the `removeLast` method that removes the item at the end of the collection, making the collection one item smaller.

```
public void removeLast() {
```

```
}
```

Question 8. Union for Sets

[14 marks]

The union of two Sets is the the set of items that are in one or other or both of the sets. Computing the union of two sets can be done efficiently by the following algorithm:

- Copy each Set to an Indexed collection,
- Sort each Indexed collection,
- Merge the two sorted collections, keeping only one copy of items that are in both collections.

Complete the following union method that performs the last step of this algorithm: given two (sorted) Indexed collections, it constructs a Set of all the items that are in either collection, using a Merge-like algorithm. The cost of your method should be $O(n)$, where n is the number of items in the two collections. Note, adding an item to the end of a SortedArraySet is $O(1)$.

```
public static Set union (Indexed a, Indexed b, Comparator test){
    // Assumes that a and b are sorted
    SortedArraySet union = new SortedArraySet(test);

    return union;
}
```

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

This page gives brief specifications of the interfaces that you may need to use in this test.

```
public interface Enumeration {
    public boolean hasMoreElements();
    public Object nextElement();
}

public interface Comparator {
    public int compare(Object o1, Object o2);
}

public interface BufferedReader{
    public String readLine();
}

public interface Collection {
    public boolean isEmpty ();
    public int size ();
    public Enumeration elements ();
}

public interface Indexed extends Collection{
    // Implementations: Vector
    public Object elementAt (int index);
    public void setElementAt (Object v, int index);
    public void addElementAt (Object val, int index);
    public void removeElementAt (int index);
}

public interface Bag extends Collection {
    // Implementations: ArrayBag, SortedArrayBag, BucketHashBag, OpenHashBag
    public void addElement (Object value);
    public boolean containsElement (Object value);
    public Object findElement (Object value);
    public void removeElement (Object value);
}

public interface Set extends Bag {
    // Implementations: ArraySet, SortedArraySet, BucketHashSet, OpenHashSet
    // operations of Bag and the following
    public void unionWith (Bag aSet);
    public void intersectWith (Bag aSet);
    public void differenceWith (Bag aSet);
    public boolean subsetOf (Bag aSet);
}

public interface Map extends Collection {
    // Implementations: ArrayMap, SortedArrayMap, BucketHashMap, OpenHashMap
    public boolean containsKey (Object key);
    public Object get (Object key);
    public void removeKey (Object key);
    public void set (Object key, Object value);
}

public interface SortAlgorithm {
    // Implementations: MergeSort, InsertionSort, SelectionSort, BubbleSort, Partition
    // Constructors require a Comparator
    public void sort(Indexed collection);
}
```