VICTORIA UNIVERSITY OF WELLINGTON
Te Whare Wananga o te Upoko o te Ika a Maui

EXAMINATIONS — 2002

END-YEAR

COMP 202

Formal methods of Computer Science

Time Allowed: 3 Hours

Instructions: There are seven (7) questions, worth fourteen (14) marks each, making 98 marks in total.
Answer all the questions.

You may use printed foreign language dictionaries.
You may not use calculators or electronic dictionaries.

Question 1. [14 marks]

The following procedure takes an integer, a, and is intended to return $|a|$, the absolute value of a.

$$
\begin{array}{ll}
\multicolumn{2}{l}{\textbf{procedure } abs(\textbf{in } a, \textbf{out } x)} \\
 & \textbf{begin} \\
\{A_1\} & \quad \textbf{if } a < 0 \textbf{ then} \\
\{A_2\} & \qquad x := -a \\
\{A_3\} & \quad \textbf{elsif } a > 0 \textbf{ then} \\
\{A_4\} & \qquad x := a \\
\{A_5\} & \quad \textbf{fi} \\
\{A_6\} & \textbf{end}
\end{array}
$$

(a) Give the specification (signature, precondition and postcondition) that this procedure satisfies. [3 marks]

(b) Briefly outline how to verify that the program meets your specification. [4 marks]

(c) The following is (part of) a definition of a semantic function $\mathcal{V}$ for evaluating Boolean expressions:

$$
\begin{array}{lcl}
\mathcal{V}(p \textbf{ and } q) & = & \begin{cases} \mathcal{V}(q), & \text{if } \mathcal{V}(p) = 1 \\ 0, & \text{otherwise} \end{cases} \\
\\
\mathcal{V}(\textbf{not } p) & = & \begin{cases} 1, & \text{if } \mathcal{V}(p) = 0 \\ 0, & \text{otherwise} \end{cases} \\
\\
\mathcal{V}(\textbf{true}) & = & 1 \\
\\
\mathcal{V}(\textbf{false}) & = & 0
\end{array}
$$

Extend the definition to handle **Boolean variables**, v, whose values are determined by a store. **Hint:** you will need to modify the given clauses as well as adding a new one. [7 marks]

Question 2. [14 marks]

$M_1 = (Q, \Sigma, \delta, q_0, F)$, where:

- $Q = \{S_0, S_1, S_2, S_3\}$
- $\Sigma = \{0, 1\}$
- $q_0 = S_0$
- $F = \{S_3\}$

and δ is given by the table:

δ	0	1
S_0	$\{S_1, S_2\}$	$\{S_2\}$
S_1	$\{\}$	$\{S_2, S_3\}$
S_2	$\{S_3\}$	$\{S_1, S_2\}$
S_3	$\{\}$	$\{\}$

(a) Draw M_1. [4 marks]

(b) Find an FA which accepts the same language as M_1. Be sure to explain why your FA accepts the same language as the M_1. [10 marks]

Question 3. [14 marks]

(a) State Kleene's theorem. [4 marks]

(b) Show that, if L_1 and L_2 are regular languages, then so are:

1. $L_1 \cup L_2$
2. $L_1 L_2$ [5 marks]

(c) Let r and s be regular expressions. State which of the following equalities hold:

1. $\mathbf{r} + \mathbf{s} = \mathbf{s} + \mathbf{r}$
2. $\mathbf{rs} = \mathbf{sr}$
3. $\emptyset^* = \Lambda^*$
4. $\mathbf{r}^* + \mathbf{s}^* = (\mathbf{r} + \mathbf{s})^*$
5. $(\mathbf{r} + \mathbf{s})^* = (\mathbf{s}^*\mathbf{r}^*)^*$ [5 marks]

Question 4. [14 marks]

Here are four grammars.

$$
\begin{array}{llll}
G_1: & S \to AB & G_2: & S \to T \mid VC \\
 & A \to a \mid aA & & T \to B \mid aTc \\
 & B \to b \mid bB & & B \to \Lambda \mid bB \\
 & & & V \to \Lambda \mid aVb \\
 & & & C \to \Lambda \mid cC
\end{array}
$$

$$
\begin{array}{llll}
G_3: & S \to aE & G_4: & S \to SaS \mid SbS \mid c \\
 & E \to A \mid B & & \\
 & A \to aA \mid B & & \\
 & B \to bB \mid B & &
\end{array}
$$

(a) State which one(s) describe regular languages, and explain why. [2 marks]

(b) State which one(s) describe context free languages, and explain why. [2 marks]

(c) Using G_4, give two different leftmost derivations for the string *cacbc*. [2 marks]

(d) Give an unambiguous grammar, G_5, that generates the same language as G_4. [4 marks]

(e) Draw a parse tree for the string *cacbc*, using your grammar G_5. [4 marks]

Question 5. [14 marks]

Below is a CFG, G, for a subset of $\{<, >, x\}^*$ in which $<$ and $>$ are balanced like the brackets of an arithmetic expression. For example, "$<<x>>$" and "$<x<x><>>$" are in the language, but "$><$" and "$<x>><$" are not.

$$
\begin{array}{lll}
S \to \,< S' & A \to B\ A' & B \to x \mid S \\
S' \to \,> \mid A> & A' \to \Lambda \mid A &
\end{array}
$$

(a) Compute:

1. $\mathit{first}(S)$
2. $\mathit{first}(A)$
3. $\mathit{first}(A')$
4. $\mathit{follow}(A')$ [4 marks]

(b) Hence, show that G is LL(1). [3 marks]

(c) Define a recursive descent parser for the language defined by G. [7 marks]

Question 6. [14 marks]

The following grammar G defines the language $L = \{wcw^R | w \in \{a, b, \}^*\}$, where w^R is the reverse of w:

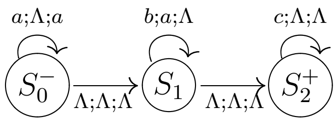
$$S \to c \mid aSa \mid bSb$$

(a) Give a leftmost derivation of $abcba$ using G. [3 marks]

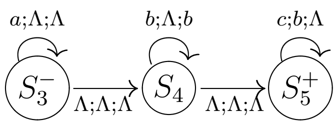
(b) Draw a PDA for L using a *bottom-up* strategy. [3 marks]

The following PDAs P_1 and P_2 define context free languages L_1 and L_2 (respectively):

P_1 :



P_2 :



(c) Describe the languages L_1 and L_2. [2 marks]

(d) Is the concatenation (product) $L_3 = L_1L_2$ context free?
If so, draw a PDA for L_3. If not, explain why not. [3 marks]

(e) Is the intersection $L_4 = L_1 \cap L_2$ context free?
If so, draw a PDA for L_4. If not, explain why not. [3 marks]

Question 7. [14 marks]

(a) Define the following terms:

1. computable language
2. computably enumerable language [5 marks]

(b) Explain what we mean when we say a problem is decidable. [4 marks]

(c) Describe the *halting problem*, and show why it is undecidable. [5 marks]

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