VICTORIA UNIVERSITY OF WELLINGTON
Te Whare Wananga o te Upoko o te Ika a Maui

EXAMINATIONS — 2002

TERM TEST

COMP 202

Formal Methods of Computer Science

Time Allowed: 50 minutes

Instructions: There are **four** (4) questions.
Each question is worth **twenty-five** (25) marks.
Answer **all** the questions.
Show **all** your working.

Question 1.

The following program takes two integers, a and b, and returns integers x and y such that x is the smaller of a and b, and y is the larger of a and b.

$$
\begin{array}{ll}
 & \textbf{begin} \\
\{A_1\} & \quad \textbf{if } a \leq b \textbf{ then} \\
\{A_2\} & \qquad x := a; \\
\{A_3\} & \qquad y := b \\
\{A_4\} & \quad \textbf{else} \\
\{A_5\} & \qquad x := b; \\
\{A_6\} & \qquad y := a \\
\{A_7\} & \quad \textbf{fi} \\
\{A_8\} & \textbf{end}
\end{array}
$$

postcondition) for this problem.

(a) Write a formal specification (signature, precondition, and postcondition) for this problem. [6 marks]

(b) Give assertions $A_1, \ldots, A_8$ that may be used to prove this program correct. **You do not need to complete the proof!** [12 marks]

(c) Explain how a **loop invariant** may be used in the verification of a while-program. [7 marks]

Question 2.

Let $M_1 = (Q, \Sigma, \delta, q_0, F)$, where

$$\begin{aligned} Q &= \{S_0, S_1, S_2, S_3\} \\ \Sigma &= \{0, 1\} \\ q_0 &= S_0 \\ F &= \{S_1\} \end{aligned}$$

δ	0	1	Λ
S_0	$\{\}$	$\{S_2\}$	$\{S_1\}$
S_1	$\{S_1\}$	$\{S_3\}$	$\{\}$
S_2	$\{\}$	$\{S_2\}$	$\{S_3\}$
S_3	$\{S_1\}$	$\{\}$	$\{\}$

(a) For each of the following strings state whether it is accepted by M_1:

1. 00
2. 10
3. 01
4. 011
5. 11011

[5 marks]

(b) *Outline* the method for constructing an NFA from an NFA with Λ transitions, such that both machines accept the same language. [10 marks]

(c) Find an NFA which accepts the same language as M_1. [10 marks]

Question 3.

The following machines M_2 and M_3 accept $\mathrm{Language}(\mathbf{1})$ and $\mathrm{Language}(\mathbf{0})$ respectively.

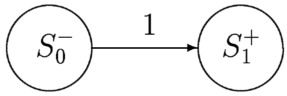

Figure 1: M_2

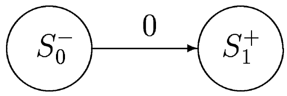

Figure 2: M_3

Draw NFA with Λ transitions which accept the following languages.

(a) $\mathrm{Language}(\mathbf{1}^*)$ [5 marks]

(b) $\mathrm{Language}((\mathbf{1}+\mathbf{0})^*)$ [5 marks]

(c) $\mathrm{Language}(\mathbf{10}+\mathbf{01})$ [5 marks]

(d) $\mathrm{Language}(\mathbf{1}+\emptyset)$ [5 marks]

(e) $\mathrm{Language}(\mathbf{0}+\mathbf{\Lambda})$ [5 marks]

Question 4.

(a) *State* Kleene's Theorem. [5 marks]

(b) The pumping lemma tells us that if L is a regular language then there is a number p, such that $(\forall s \in L)(|s| \geq p \Rightarrow s = xyz)$, where:

1. $(\forall i \geq 0) xy^i z \in L$
2. $|y| > 0$
3. $|xy| \leq p$

Outline the proof of the pumping lemma. [10 marks]

(c) Using the pumping lemma, or otherwise, show that the language $\{1^n 0^n | n \geq 0\}$ is not regular. [10 marks]

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