TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

VICTORIA
UNIVERSITY OF WELLINGTON

EXAMINATIONS — 2005

END-YEAR

COMP 202
Formal Methods
of Computer Science

Time Allowed: 3 Hours

Instructions: There are seven (7) questions, worth fourteen (14) marks each, making ninety-eight (98) marks in total.
Answer all the questions.

You may use printed foreign language dictionaries.
You may not use calculators or electronic dictionaries.

Question 1.

(a) Describe the main idea behind the construction of a finite automaton from a nondeterministic finite automaton, such that both machines define the same language. [5 marks]

(b) Show that every finite language is regular. [3 marks]

(c) Let M_1 be the nondeterministic finite automaton with Λ transitions defined by $M_1 = (Q, \Sigma, \delta, q_0, F)$, where:

- $Q = \{S_0, S_1, S_2\}$
- $\Sigma = \{a, b\}$
- $q_0 = S_0$
- $F = \{S_2\}$

and δ is given by the table:

δ	a	b	Λ
S_0	$\{S_0\}$	$\{\}$	$\{S_1\}$
S_1	$\{\}$	$\{S_1\}$	$\{S_2\}$
S_2	$\{S_2\}$	$\{\}$	$\{S_2\}$

(i) Draw M_1.

(ii) Find a nondeterminstic finite automaton without Λ transitions which accepts the same language as M_1. [6 marks]

Question 2.

(a) Let M_1 and M_2 be nondeterministic finite automata with Λ transitions, and let $L_1 = \text{Language}(M_1)$ and $L_2 = \text{Language}(M_2)$.

Draw nondeterministic finite automata with Λ transitions which describe the following languages. Your diagrams should indicate how the states of your machines relate to the states of M_1 and M_2.

(i) $L_1 \cup L_2$

(ii) $L_1 L_2$

(iii) $L_1 \cap \overline{L_1}$ [6 marks]

(b) Let: $\Sigma = \{a, b\}$
$L_1 = \text{Language}(\mathbf{a}(\mathbf{a} + \mathbf{b})^*)$
$L_2 = \text{Language}((\mathbf{ba})^*)$
$L_3 = \text{Language}(\mathbf{a}^* + \mathbf{b}^*)$

Give a string which:

(i) is in L_1 but not in L_2 or L_3

(ii) is in L_2 but not in L_1 or L_3

(iii) is in L_3 but not in L_1 or L_2 [3 marks]

(c) State Kleene's theorem. [5 marks]

Question 3.

Consider the context free grammar

$$\begin{array}{ll} (1,2) & A \to aA \mid B \\ (3,4,5) & B \to aAbA \mid C \mid \Lambda \\ (6) & C \to b \end{array}$$

(a) Define Σ, N, and S for this grammar. [2 marks]

(b) Draw a parse tree for the string *aabb*. [2 marks]

(c) Show that the grammar is ambiguous. [3 marks]

(d) List the **nullable nonterminals**. [1 mark]

(e) List the **unit productions**. [1 mark]

(f) Find an equivalent grammar with no unit productions. [5 marks]

Question 4.

Consider the context free grammar

$$\begin{array}{rl} (1,2) & S \rightarrow TSb \mid T \\ (3,4,5,6) & T \rightarrow aUa \mid bUb \mid a \mid b \\ (7,8) & U \rightarrow S \mid cTU \end{array}$$

(a) Explain what it means for a grammar to be in LL(1) form. [3 marks]

(b) Show that the above grammar is **not** in LL(1) form. [2 marks]

(c) Rewrite the grammar so that it is in LL(1) form. [4 marks]

(d) Complete the *ParseU* procedure whose heading appears below, for a recursive-descent parser to recognize the *U* nonterminal. You may assume that procedures *ParseS* and *ParseT* for recognizing the other nonterminals are already written. The parameter *ss*, providing the input for the parser, is a sequence of terminal symbols. You do not need to return a parse tree.

procedure *ParseU* (**in out** *ss*)
begin
  . . .
end

[5 marks]

Question 5.

(a) Define a pushdown automaton that accepts each of the following languages:

(i) The language consisting of strings over the alphabet $\{a, b\}$ with the same number of as and bs, but occurring in any order

(ii) The language $\{a^m b^n c^{m-n} \mid m \geq n > 0\}$ [6 marks]

(b) Define a pushdown automaton that accepts the language generated by the following grammar, using a **bottom-up** (shift-reduce) strategy.

$$\begin{aligned} S &\rightarrow TU \mid \Lambda \\ T &\rightarrow aSa \mid Sb \\ U &\rightarrow aU \mid a \end{aligned}$$

[4 marks]

(c) Briefly outline a proof that any language accepted by a pushdown automaton may be generated by a context free grammar. [4 marks]

Question 6.

(a) A Turing machine may be defined by a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$.

Explain each of the following:

(i) Σ

(ii) Γ

(iii) δ [3 marks]

(b) Let L_1 and L_2 be regular languages. Is it possible to write a program which decides whether L_1 and L_2 are the same language? Justify your answer. [4 marks]

(c)

(i) Define "universal Turing machine".

(ii) Briefly outline how a Turing machine may be encoded so as to provide the input for another Turing machine.

(iii) Discuss the relevance of the existence of universal Turing machines in proving that there are undecidable problems. [7 marks]

Question 7.

You are given strings s and t, both of length n ($|s| = |t| = n$), and are required to determine whether s is the reverse of t ($s = t^R$).

(a) Give a formal specification (signature, precondition, and postcondition) for the problem. [3 marks]

(b) State three requirements for proving the correctness of a loop in a while-program using a loop invariant. [3 marks]

(c) The following fragment of program partly satisfies the specification: it is possible to determine from the final value of i whether $s = t^R$.

begin
1 $i := n - 1$;
2 $j := 0$;
3 **while** $i \geq 0$ **and** $s[i] = t[j]$ **do**
4     $i := i - 1$;
5     $j := j + 1$
6 **end**

An invariant that may be used to prove the loop correct is

$$-1 \leq i < n \wedge j = n - i - 1 \wedge (\forall k \in 0..j-1)(s[k] = t[n-k+1])$$

(i) Show that the loop invariant holds at the start of the loop.

(ii) State an assertion that holds at the end of the loop, and show that it holds there.

(iii) Hence, give statements that can follow the loop (between lines 5 and 6) to complete the program, and show that your postcondition from part (a) is satisfied. [8 marks]

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