TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

EXAMINATIONS — 2007

MID-TERM TEST

COMP 202
Formal Methods of Computer Science

Time Allowed: 90 minutes

Instructions: There are **five** (5) questions.
Each question is worth **eighteen** (18) marks.
Answer **all** the questions.
Show **all** your working.

Question 1.

Consider the following informal description of a problem:

> Given strings s and t, return *true* iff s is a prefix (initial segment) of t.

(a) Give a formal specification (signature, precondition, and postcondition) for the problem. Clearly state any assumptions you make about the problem, and give clear descriptions (in English or formal) of any notation you introduce. [6 marks]

(b) Define an iterative program in the imperative language which satisfies your specification. Use $s[i]$ to access the ith element of a string s (counting from 0), and $|s|$ to determine the length of s. [6 marks]

(c) Define a recursive program in the applicative language which satisfies your specification. Use *head* and *tail* to access the parts of a string, and *empty* to determine if a string is empty. [6 marks]

Question 2.

(a) *State* Kleene's Theorem. [4 marks]

(b) *Outline* the method for constructing an FA from an NFA with Λ transitions, such that both machines accept the same language. [6 marks]

(c) Let $M_1 = (Q, \Sigma, \delta, q_0, F)$, where

$$\begin{aligned} Q &= \{S_0, S_1, S_2, S_3\} \\ \Sigma &= \{a, b\} \\ q_0 &= S_0 \\ F &= \{S_1\} \end{aligned}$$

δ	a	b	Λ
S_0	$\{S_0, S_1\}$	$\{S_2\}$	$\{\}$
S_1	$\{\}$	$\{S_3\}$	$\{S_2\}$
S_2	$\{S_1, S_3\}$	$\{\}$	$\{\}$
S_3	$\{S_2\}$	$\{\}$	$\{S_1, S_2\}$

(i) Draw M_1. [2 marks]

(ii) Find an FA which accepts the same language as M_1. [6 marks]

Question 3.

(a) Given an alphabet $\Sigma = \{a, b\}$, draw NFA with Λ transitions which accept the following languages:

(i) Language($\mathbf{ba}^*$) [3 marks]

(ii) Language($\mathbf{b} + \mathbf{a}$) [3 marks]

(iii) Language($(\mathbf{a} + \mathbf{b})^*(\mathbf{b}^*\mathbf{a}^*)^*$) [3 marks]

(iv) Language($(\mathbf{b} + \mathbf{a})\mathbf{b}(\mathbf{b} + \mathbf{a})^*$) [3 marks]

(b) Let $M_2 = (Q, \Sigma, \delta, q_0, F)$, where

$$\begin{aligned} Q &= \{S_1, S_2, S_3\} \\ \Sigma &= \{a, b\} \\ q_0 &= S_1 \\ F &= \{S_1, S_3\} \end{aligned}$$

δ	a	b
S_1	S_3	S_2
S_2	S_3	
S_3	S_2	S_1

(i) Draw M_2. [2 marks]

(ii) Describe the language accepted by M_2. [4 marks]

Question 4.

Let $M_3 = (Q, \Sigma, \delta, q_0, F)$, where

$$\begin{aligned} Q &= \{A, B, C\} \\ \Sigma &= \{a, b\} \\ q_0 &= A \\ F &= \{B\} \end{aligned}$$

δ	a	b
A	B	C
B	$-$	B
C	A	B

(a) Give a regular grammar that generates the language accepted by M_3. [4 marks]

(b) Define a recursive scanner that recognizes the language accepted by M_3. [6 marks]

(c) Give a regular expression that describes the language accepted by M_3. [8 marks]

Question 5.

(a) The pumping lemma tells us that if L is a regular language then there is a number p, such that $(\forall s \in L)(|s| \geq p \Rightarrow s = xyz)$, where:

1. $(\forall i \geq 0) xy^i z \in L$
2. $|y| > 0$
3. $|xy| \leq p$

(i) *Outline* the proof of the pumping lemma. [6 marks]

(ii) Give an example of a non-regular language, and use the pumping lemma to show that it is non-regular. [6 marks]

(b) *Explain* the role of Kleene's theorem in showing that the class of regular languages is closed under union, concatenation, Kleene closure, and complement. [6 marks]

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