School of Mathematical and Computing Sciences
COMP 202 : Formal Methods of Computer Science

Test (26 August, 1998)

Time allowed = 90 minutes

Total marks = 50

Answer ALL questions

1. A string s is a *common prefix* of strings t and u iff s is a prefix of t and also a prefix of u; i.e. $t = st'$ and $u = su'$ for some strings t' and u'.

   s is the *longest common prefix* of t and u iff s is a common prefix of t and u and there is no string s' such that s' is a common prefix of t and u and $|s'| > |s|$.

   (a) Prove the following properties of common prefixes:

   (i) Every pair of strings s and t has at least one common prefix. [2 marks]

   (ii) If s is a common prefix of t and u, and v is a common prefix of $s\backslash t$ and $s\backslash u$, then $s \frown v$ is a common prefix of t and u. [4 marks]

   (b) Explain how the above properties can be used to construct an algorithm to find the longest common prefix of two strings, s and t.

   Write the resulting algorithm as a functional program, using *head*, *tail*, $NULL$ and $\frown$ to access and manipulate strings. [6 marks]

2. (a) Write a Regular Expression to describe the language consisting of all strings over $\{a, b, c\}$ which contain *either* exactly one a or exactly two bs.

   For example, a, bb, bab, $ccbbbabc$ and $acacbcacba$ are in this language, but λ, b, aba and ccc are not. [4 marks]

   (b) Write a Regular Expression to describe the language consisting of all strings over $\{a, b, c\}$ for which all of the following conditions hold:

   - all of the as and bs occur before all of the cs,
   - if the first a occurs *before* the first b, there must be an *even* number of cs, and
   - if the first a occurs *after* the first b, there must be an *odd* number of cs.

   For example, λ, a, b, c, cc, ac, bcc, $abcc$, bac, $abaaccccccc$ and $bbaabccc$ are in this language, but ba, abc, $abccc$, $bacc$ and $bababababcccc$ are not. [4 marks]

3. Consider the regular expression $(b|c)^*(a(b|c)(b|c)^*)^*a^*$.

   Draw a transition diagram for the NFA obtained by applying the "top down" construction (as described in the Course Notes).

   Make sure you show *all* states and edges given by the construction! [6 marks]

4. Consider the NFA $M = (\{1,2,3,4,5,6,7,8\}, 1, \{a,b,c\}, N, \{8\})$, where N is defined by the following transitions:

   $(1,a;1), (1,\epsilon;2), (1,\epsilon;3), (2,b;4), (2,c;5), (3,b;5), (3,c;4),$

   $(4,a;6), (5,a;7), (6,\epsilon;8), (7,\epsilon;8), (8,\epsilon;2), (8,\epsilon;3)$

   (a) Draw a transition diagram depicting this NFA. [2 marks]

   (b) Give a trace showing the behaviour of the the NFA with *aabaca* as input.
   Be sure to show *all* states the NFA could be in at each step. [4 marks]

   (c) Construct an equivalent DFA, using the subset construction.
   Show the relationships between the states in your DFA and those of the NFA. [6 marks]

   (d) Give the equations relating the regular expressions denoting the $From$ sets for the states in your DFA.
   Solve these equations to obtain a regular expressions denoting the $From$ set for each state in your DFA. [6 marks]

5. Consider the language L containing all strings over $\{a,b,c\}$, in which there are an equal number of as and bs.

   Prove that L is not regular, using the Myhill-Nerode Theorem. [6 marks]