Family Name:.............................. Other Names: ....

Student ID:.............................. Signature ....

COMP 261 : Test 1

2 April 2026 ** WITH SOLUTIONS **

Instructions

- Time allowed: **120 minutes**
- Attempt **all** the questions. There are 120 marks in total.
- Write your answers in this test paper and hand in all sheets.
- If you think a question is unclear, ask for clarification.
- This test contributes 30% of your final grade.
- You may use dictionaries and calculators.
- You may write notes and working on this paper, but make sure your answers are clear.

Questions

Questions	Marks	
1. Grammars and Parse Trees	[17]	
2. Coding a Parser	[23]	
3. Ambiguous Grammars	[20]	
4. Minimum Spanning Trees	[20]	
5. Disjoint Sets (Union-Find)	[12]	
6. Network Flow Analysis	[28]	
	TOTAL:	

Question 1. Grammars and Parse Trees **[17 marks]**

Consider the following grammar that describes a simple language for recording sensor readings. In this grammar

- Non-terminals are in uppercase; terminals are enclosed in quotation marks,
- `[...]+` means one or more repetitions of what is in the brackets.
- `NUM` matches any terminal that is a non-negative integer.

```
LOG ::= [ RECORD ]+
RECORD ::= "(" ID ":" VALUES ")"
ID ::= "sensor" NUM
VALUES ::= NUM [ "," NUM ]+
NUM ::= matches "[0-9]+"
```

Please note that space is used as the delimiter in Scanner.

(a) **[12 marks]**

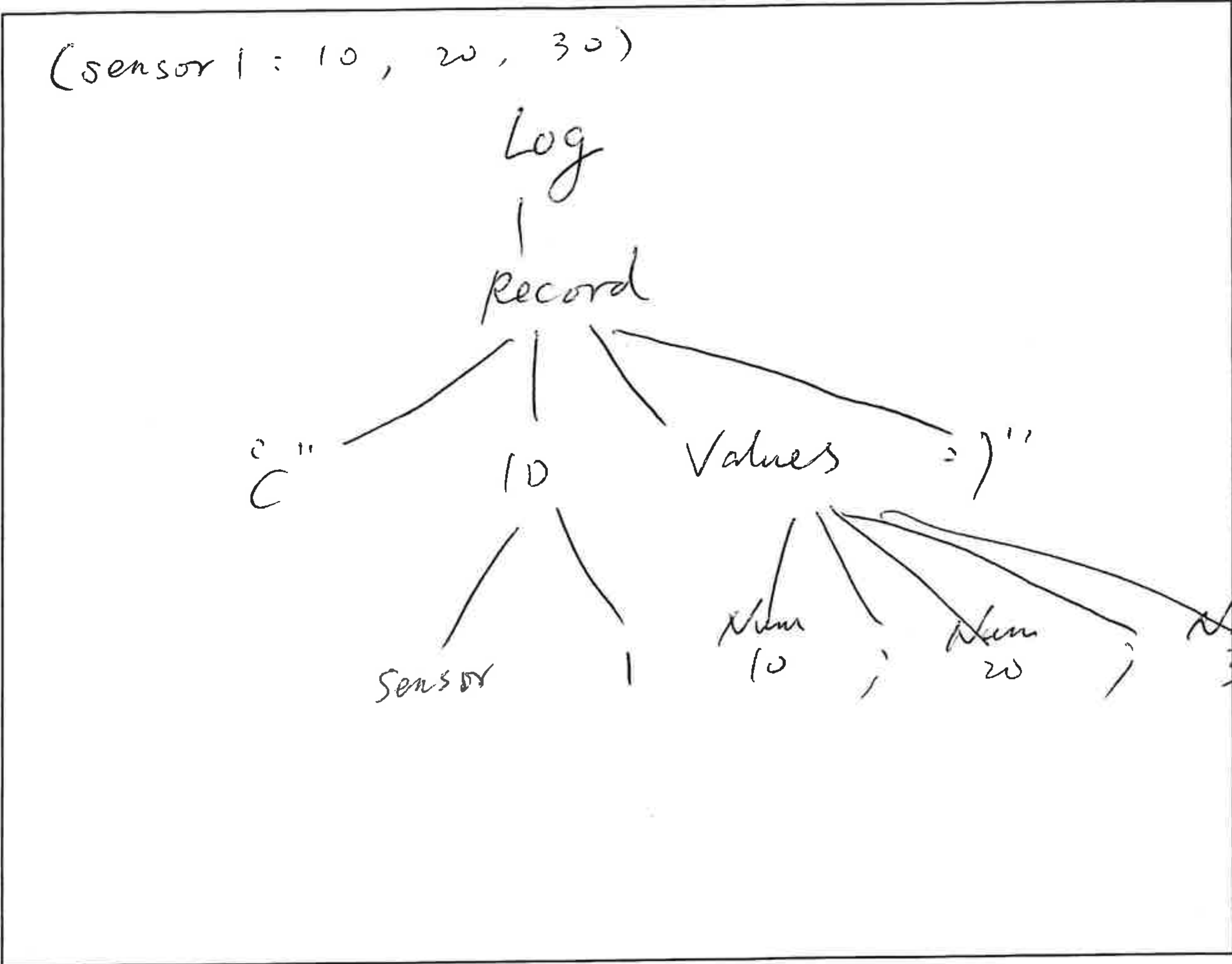
Determine whether each sentence is valid or invalid according to the grammar above, and circle either "valid" or "invalid."

valid / invalid :	`[ sensor 1 :  1 , 2 , 3 ]`
valid / invalid :	`( sensor 1 :  10 , 20 , 30 )`
valid / invalid :	`( sensor 4 :  5 3 2 )`
valid / invalid :	`( ( sensor 5 :  1 , 2 ) )`
valid / invalid :	`( sensor 3 :  8 , 9 ) , ( sensor 4 :  10 , 11 )`
valid / invalid :	`( sensor 2 :  3 ) ( sensor 1 :  2 )`

(Question 1 continued on next page)

(Question 1 continued)

(b) **[5 marks]** Select a **valid** sentence from (a) and draw its Concrete Parse Tree according to the grammar. If none of them is valid, you may write your own sentence and draw its tree.



Question 2. Coding a Parser **[23 marks]**

Suppose you are writing a parser for the grammar in Question 1.

```
LOG ::= [ RECORD ]+
RECORD ::= "(" ID ":" VALUES ")"
ID ::= "sensor" NUM
VALUES ::= NUM [ "," NUM ]+
NUM ::= matches "[0-9]+"
```

Your parser program (below) already includes some constants, three parse methods, and utility methods (require(..) and fail(..)).

The two sub-questions are:

- (a) to write more parse methods so that the top parse method should return an Abstract Syntax Tree of TNodes, or throw an exception if a sentence is invalid. We provide methods for parseLog(..), parseID(..), and parseNum(...), please read them carefully.
- (b) to write the LogNode, RecordNode classes defining the different kinds of TNodes.

You are to complete the methods in Java, not pseudocode.

```
//-------- constants (patterns) --------------------
static final Pattern LEFT_PAT = Pattern.compile("\\(");
static final Pattern RIGHT_PAT = Pattern.compile("\\)");
static final Pattern NUM_PAT = Pattern.compile("[0-9]+");
static final Pattern COMMA_PAT = Pattern.compile("\\,");
static final Pattern COLON_PAT = Pattern.compile("\\:");
static final Pattern SENSOR_PAT = Pattern.compile("sensor");

//-------- parse... methods --------------------
/** Parses the rule: LOG ::= [ RECORD ]+ */
public TNode parseLog(Scanner s){
    List<TNode> logs = new ArrayList<TNode>();
    do {
            logs.add(parseRecord(s));
    } while (s.hasNext());
    return new LogNode(logs);
}
/** Parses the rule: ID ::= "sensor" NUM        Please note it returns an integer */
public int parseID(Scanner s){
    require(SENSOR_PAT, s);
    int num = parseNum(s);
    return num;
}
public int parseNum(Scanner s){
    if (s.hasNext("[0-9]")) {return s.nextInt();}
    fail("Expecting integer"); return -1;
}
//-------- Utility methods --------------------
public void require(Pattern pat, Scanner s){
    if (s.hasNext(pat)) {s.next(); return;}
    fail("expecting "+ pat);
}
public void fail(String msg){ System.out.println(msg); throw new RuntimeException(msg);}
```

(Question 2 continued on next page)

(Question 2 continued)

(a) **[13 marks]** Complete the parseRecord(..), parseValues(..) methods below.

```
/** Parses the rule:  RECORD ::= "(" ID ":"  VALUES ")"          */
public TNode parseRecord(Scanner s){

        require(LEFT_PAT,s);
        int id = parseID(s);
        require(COLON_PAT,s);
        List<Integer> values = parseValues(s);
        require(RIGHT_PAT, s);
        return new RecordNode(id, values);

}
/** Parses the rule:  VALUES ::= NUM [ "," NUM ]+   */
/** Please note it returns an Integer List          */
public List<Integer> parseValues(Scanner s){

        List<Integer> values = new ArrayList<Integer>();

        values.add(parseNum(s));

        do {
                require(COMMA_PAT,s);
                values.add(parseNum(s));
        } while(s.hasNext(COMMA_PAT));

        return values;

}
```

(b) **[10 marks]** Define the TNode classes

The parser in the previous sub-question needs the LogNode, RecordNode classes to create the Abstract Syntax Tree of TNodes. Suppose we want to print out "the total value of the sensor readings" for a valid sentence, for example,

```
Parser parser = new Parser();
Scanner s = new Scanner("( sensor 2 : 4 , 3 ) ( sensor 6 : 2 , 5 , 4 , 1 )")
TNode tree = parser.parseLog(s);
if (tree!=null) {System.out.println(tree.getSensorTotal());}
```

should print out 19, which is calculated as: 4 + 3 + 2 + 5 + 4 + 1

You will need to define a getSensorTotal() method for each kind of TNodes.

The TNode interface is provided as follows:

```
interface TNode{
        public int getSensorTotal();
}
```

Grammar copied from previous question:

```
LOG ::= [ RECORD ]+
RECORD ::= "(" ID ":" VALUES ")"
ID ::= "sensor" NUM
VALUES ::= NUM [ "," NUM ]+
NUM ::= matches "[0-9]+"
```

```
//-------- Node classes -------------------
class LogNode implements TNode{






    final List<TNode> records;

        public LogNode(List<TNode> records){this.records = records;}

        public int getSensorTotal(){
                int total=0;
                for (TNode r: records) {
                        total = total + r.getSensorTotal();
                }
                return total;
        }




}
class RecordNode implements TNode{






    final int id;
        final List<Integer> values;

        public RecordNode(int id, List<Integer> values){this.id=id; this.values = values;}

        public int getSensorTotal(){
                int total = 0;
                for (int n: values) {
                        total = total + n;
                }
                return total;
        }


}
```

Question 3. Ambiguous Grammars **[20 marks]**

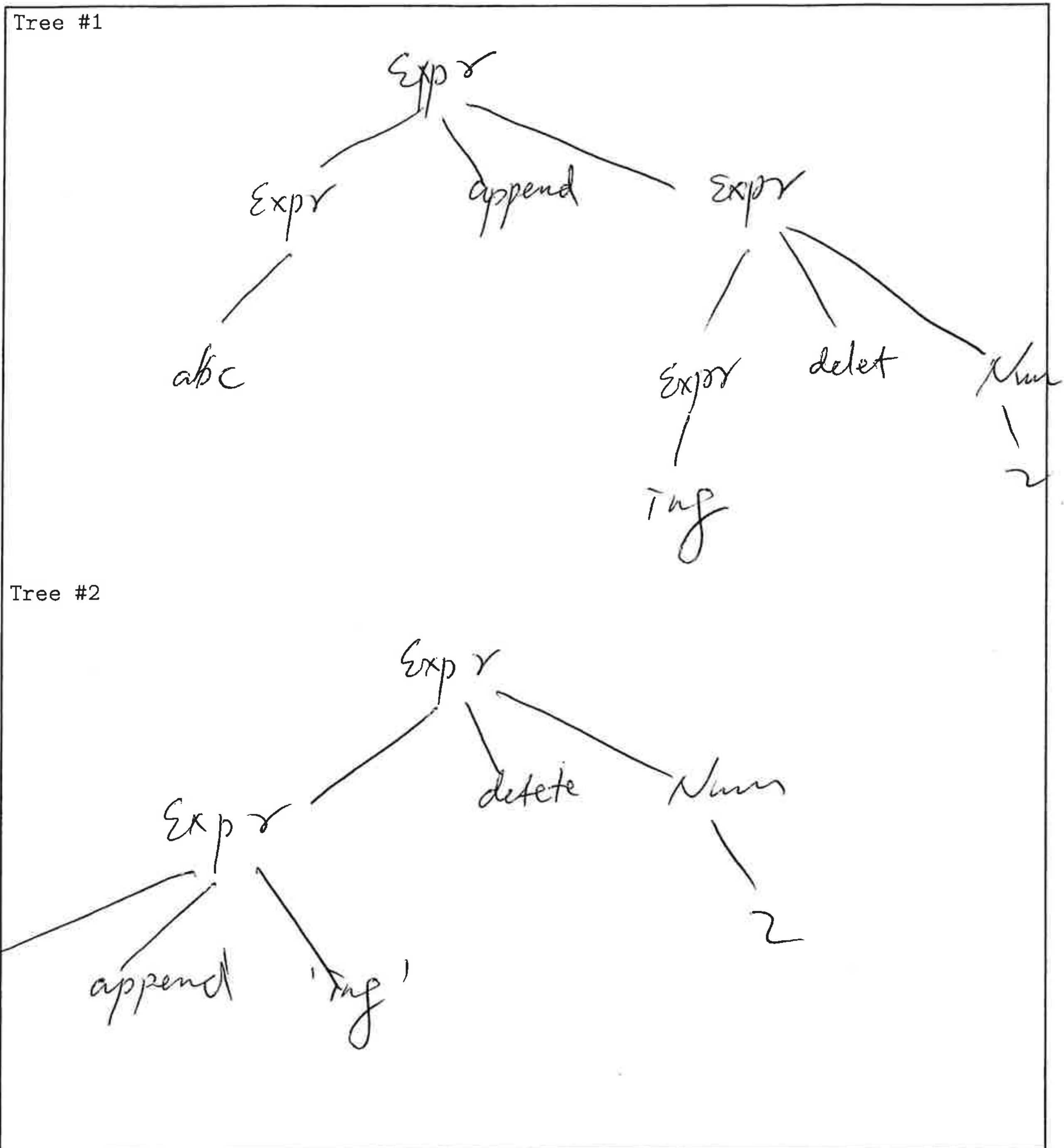
Ambiguous grammars (where a text can have multiple different parse trees) cannot be parsed by deterministic top-down recursive descent parsers, like the parsers described in the lectures.

Consider the following grammar for a simple string manipulation language:

```
EXPR ::= EXPR "append" EXPR | EXPR "delete" NUM | STR
NUM ::= matches "[0-9]+"
STR ::= matches "\"[a-z]+\""
```

(a) **[8 marks]** Draw two alternative parse trees of the following expression according to this grammar:

```
"abc" append "ing" delete 2
```



(Question 3 continued on next page)

(Question 3 continued)

(b) **[6 marks]** Rewrite the grammar to remove ambiguity by ensuring that `delete` has higher precedence than `append`.

For example,

`"aaa" append "bbb" delete 1`

should be

`"aaa" append ("bbb" delete 1).`

```
 EXPR::- TERM| TERM "append" EXPR
TERM::= STR | TERM "delete" NUM
```

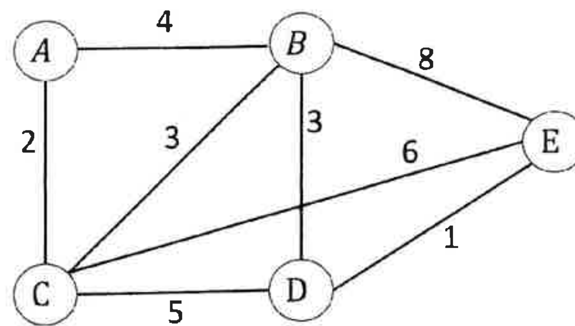
(c) **[6 marks]** Rewrite the grammar in (b) to use **right recursion only** and make sure it can be parsed by a one-token lookahead recursive descent parser, such as those introduced in the lectures.

```
EXPR::- TERM RestEXPR
RestEXPR ::- 'append' EXPR | ∈


TERM ::= STR| RestTERM
RestTERM ::- 'delete', NUM, RestTERM | ∈
```

Question 4. Minimum Spanning Trees **[20 marks]**

Consider the following weighted undirected graph.



(a) **[8 marks]** Use **Prim's Algorithm** to find the Minimum Spanning Tree (MST), starting at node **A**. List the edges in the exact order they are added to the tree. If there is a tie in edge weights, pick the node that comes first alphabetically (e.g., choose B over C).

Note: Please represent the edge you choose at each step as (X,Y), where X and Y are the two nodes connected by the edge, and represent the current tree you generate as the set of added tree nodes, e.g., {A,C,B}.

1. Start A. Candidates: (A,B,4), (A,C,2). Pick (A,C). Tree: A,C
2. Candidates: (A,B,4), (C,B,3), (C,D,5), (C,E,6). Pick (C,B). Tree: A,C,B
3. Candidates: (A,B-visited), (B,D,3), (B,E,8), (C,D,5), (C,E,6). Pick (B,D). Tree: A,C,B,D
4. Candidates: (B,E,8), (C,E,6), (D,E,1). Pick (D,E). Tree: A,C,B,D,E

Edges: (A,C), (C,B), (B,D), (D,E)

(b) **[3 marks]** What is the total cost (sum of weights) of the MST?

Total cost: 2 + 3 + 3 + 1 = 9.

(c) **[9 marks]** State whether each of the following statements is True or False. For each answer, give a one-sentence justification.

1. If a graph has unique edge weights, the MST is unique.

   True — with distinct edge weights every cut has a unique minimum crossing edge, so both cut and cycle properties force the same choices and the MST is unique.

2. Kruskal's algorithm adds edges in increasing order of weight, provided they do not form a cycle.

   True — Kruskal examines edges in nondecreasing weight order and adds an edge only if it does not create a cycle, so it builds an MST by the standard greedy rule.

3. An edge with the heaviest weight in a cycle is never part of any MST.

   True — by the cycle property the maximum-weight edge on any cycle can be removed without increasing the minimum spanning tree cost, so such a heaviest edge is never required in any MST.

Question 5. Disjoint Sets (Union-Find) **[12 marks]**

We are managing a collection of sets using the **Union-Find** data structure with the following pseudo-code for the Union operation. Initially, we have 6 isolated items (single-element trees): $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$. Each item is a root of its own tree with depth 0.

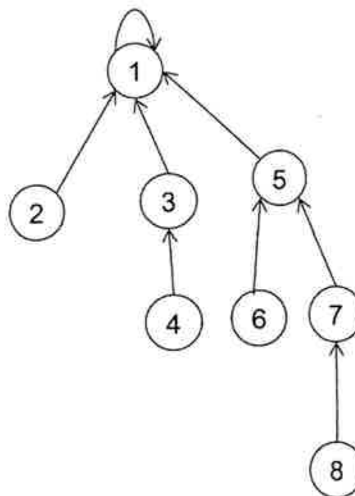
The pseudo-code for Union is:

```
Union(x, y):
  rootX = Find(x)
  rootY = Find(y)
  if rootX == rootY: return
  if rootX.depth < rootY.depth:
      rootX.parent = rootY
  else:
      rootY.parent = rootX
      if rootX.depth == rootY.depth:
          rootX.depth++
```

(a) **[8 marks]** Draw the forest (collection of trees) that results after the following sequence of operations:

1. `Union(1, 2)`
2. `Union(3, 4)`
3. `Union(5, 6)`
4. `Union(2, 4)`

(Note: When unions occur between roots of equal depth, the first argument becomes the parent of the second argument based on the code above).



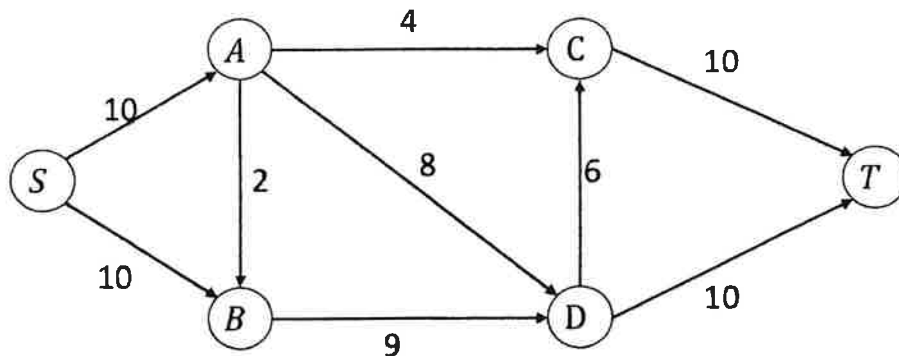
Root 1 (depth 2) →children 2, 3;
Node 3 (depth 1) →child 4.
Root 5 (depth 1) →child 6.

(b) **[4 marks]** In the resulting structure, what is the value returned by `Find(4)` (i.e., who is the representative root of 4)? How many "hops" (edges) are traversed to find it?

Answer: Root is 1. Hops: 4 →3 →1 (2 hops).

Question 6. Network Flow Analysis **[28 marks]**

Consider the following directed flow network with Source **S** and Sink **T**. The numbers on the edges represent their **Capacities**.



(a) **[6 marks]** A student proposes a flow assignment where the flow on edge $A \to D$ is **9 units**. Is this a valid flow? Explain why or why not by referring to the specific constraint that applies.

No.
Capacity constraint violation.
Capacity(A→D) = 8.
Flow(9) is greater than Capacity(8).

(b) **[6 marks]** Suppose we measure the flows around Node **D** and find the following values:

- Flow entering D: $f(A \to D) = 5$, $f(B \to D) = 6$
- Flow leaving D: $f(D \to C) = 4$, $f(D \to T) = 8$

Does this assignment satisfy the **Flow Conservation** (Balance) property for Node D? If not, explain why.

Total In = 5 + 6 = 11. Total Out = 4 + 8 = 12.
11 != 12. Conservation violated. There is a net deficit of 1 unit (more leaving than entering), which is impossible for an internal node.

(c) **[8 marks]** We now run the **Edmonds-Karp algorithm** (which uses BFS to find augmenting paths) on the network (starting with 0 flow everywhere). Suppose the first path found by BFS is:

$$S \to A \to D \to T$$

- What is the **bottleneck capacity** of this path?
- List the **residual capacity** of edge $A \to D$ (forward) and the capacity of the new backward edge $D \to A$ after this augmentation.

Path caps: S→A(10), A→D(8), D→T(10).
Bottleneck = 8.
Residual A→D: 8 - 8 = 0.
Backward D→A: 0 + 8 = 8.

(d) **[8 marks]** After the first augmentation in part (c), the algorithm continues.

1. Suppose the **second** shortest augmenting path found by BFS in the residual graph is:

$$S \to A \to C \to T$$

please calculate the amount of flow that can be pushed using this path.

Path: S →A →C →T
Bottleneck: min(Residual(S→A)=2, Capacity(A→C)=4, Capacity(C→T)=10) = 2.
Flow pushed: 2.

2. Identify the **third** shortest augmenting path found by BFS (after the second one is processed) and the amount of flow pushed.

Path: S →B →D →T
(Note: D→T has 2 remaining capacity from first step. 10 - 8 = 2).
Bottleneck: min(Capacity(S→B)=10, Capacity(B→D)=9, Residual(D→T)=2) = 2.
Flow pushed: 2.

**************************************END OF TEST 1 **
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