Family Name: ....
Other Names: ....

Signature: ....
ID Number: ....

COMP 261 Test

18 April 2016

Instructions

- Time allowed: **45 minutes** .
- Answer **all** the questions. There are 45 marks in total.
- Write your answers in the boxes in this test paper and hand in all sheets.
- If you think some question is unclear, ask for clarification.
- This test contributes 20% of your final grade
- You may use paper translation dictionaries, and non-programmable calculators.
- You may write notes and working on this paper, but make sure your answers are clear.

Questions	**Marks**	
1. Graphs	[20]	
2. Minimum Spanning Trees	[10]	
3. Graphics	[15]	
	TOTAL:	

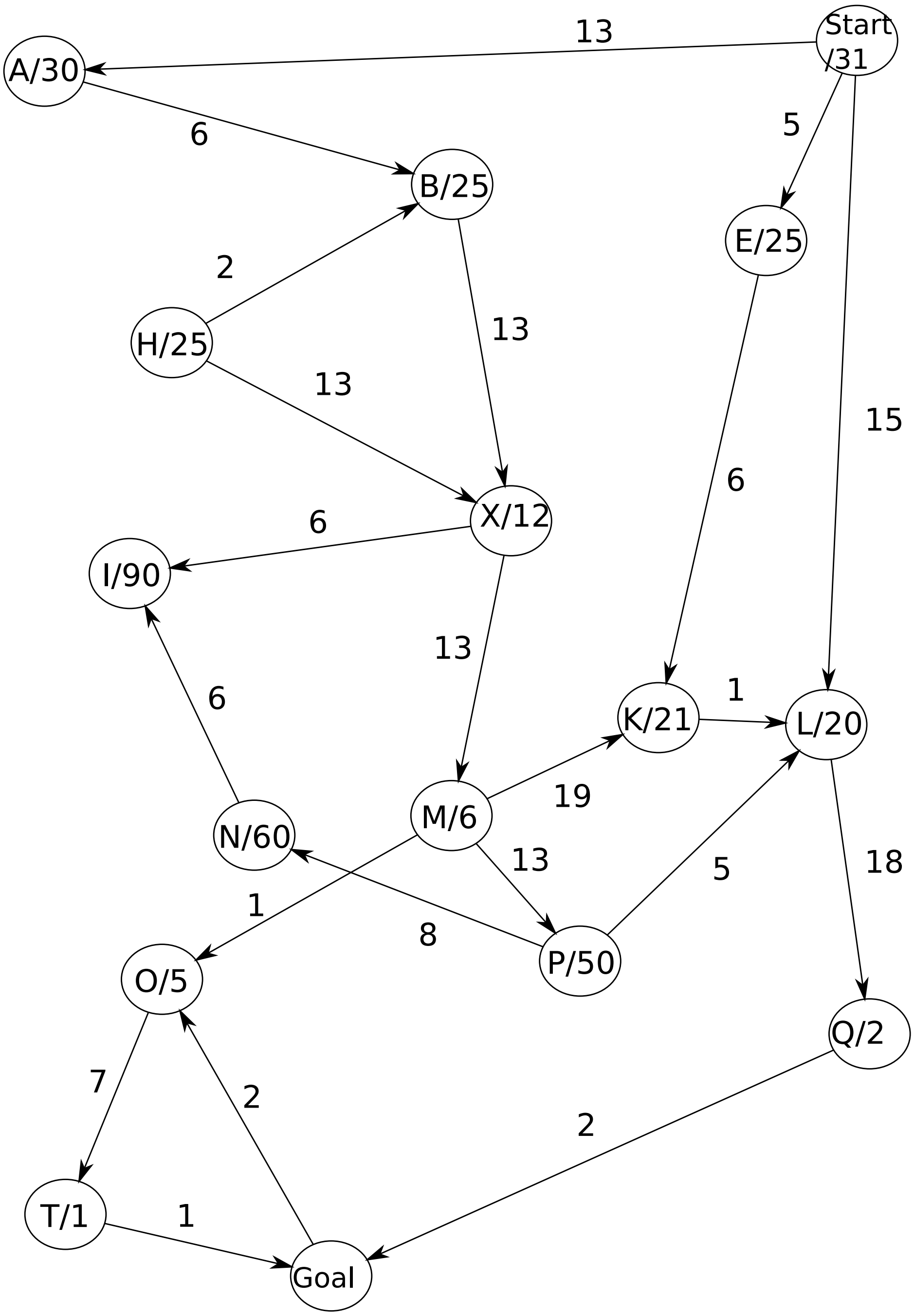
Question 1. Graphs [20 marks]

(a) [5 marks] Draw an adjacency list representation for the directed graph on the facing page (you will also find this graph on the last page of this test that can be torn off for your convenience).

Start
A
B
E
H
X
I
N
M
K
L
O
P
Q
T
Goal

(Question 1 continued on next page)

(Question 1 continued)



(Question 1 continued on next page)

(Question 1 continued)

(b) [5 marks] Given the same graph as in the part (a) show how Dijkstra's single source shortest path algorithm adapted to searching for the shortest path to a given goal node (i.e. it stops when it reaches the goal) finds the shortest path from Start to Goal.

Below, you should show the queue and the solution:

- list each element that is added to the fringe, showing the node, from-node, and the priority value;
- add neighbours in alphabetical order;
- remove nodes with the same priority value *in alphabetical order*;
- indicate the order the elements are removed from the fringe (e.g. by numbering them);
- list the nodes of the shortest path found.

Node Name:
From Node:
Priority Value:
Order Removed:

Final Path:

(Question 1 continued on next page)

(Question 1 continued)

(c) [5 marks] Given the same graph as in the parts (a) and (b) show how A* Search finds the shortest path from Start to Goal. The heuristic estimates are provided for you as values inside each node and you can assume they are admissible and consistent.

Below, you should show the queue and the solution:

- list each element that is added to the fringe, showing the node, from-node, and the priority value;
- add neighbours in alphabetical order;
- remove nodes with the same priority value *in alphabetical order*;
- indicate the order the elements are removed from the frindge (e.g. by numbering them);
- list the nodes of the shortest path found.

Node Name:
From Node:
Length So Far:
Priority Value:
Order Removed:

Final Path:

(Question 1 continued on next page)

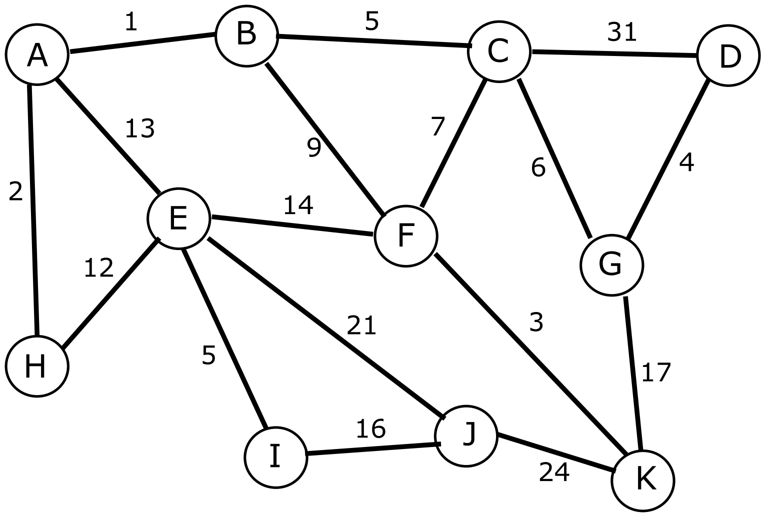
(Question 1 continued)

(d) [2 marks] Can A* Search or variation of it be run on a graph with a non-admissible and non-consistent heuristic? If not, then *what will go wrong*?

(e) [3 marks] Can A* Search or variation of it be run on a graph with a non-consistent heuristic? If it can be, then *what modification would you need to make to the fast A* Search as used in the lectures*?

Question 2. Minimum Spanning Trees [10 marks]

(a) [5 marks] Given the following graph show the steps Prim's algorithm would take to find a MST. In particular, list the edges (by stating the two nodes that they connect and the edge weight: e.g. "EF 4" for the edge between E and F with weight 4) in order that they are added to the MST. To make the answer unique and easy to mark, list the nodes in each edge in *alphabetical* order (DE not ED) and add the edges with the same weight by starting with the edge that comes first alphabetically (the word would appear first in the dictionary). Start at node A.



Alphabet:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

(Question 2 continued on next page)

(Question 2 continued)

(b) [5 marks] Given the same graph show the steps Kruskal's algorithm would take to find a MST. In particular, list the edges in order that they are added to the MST.

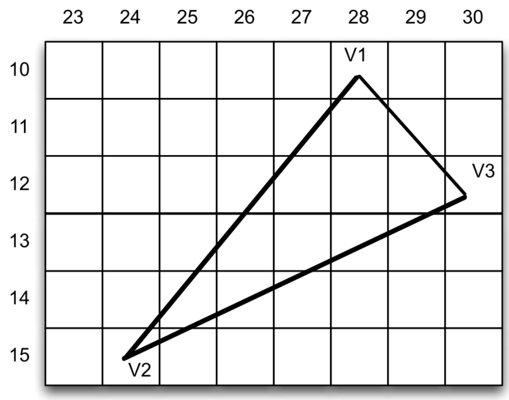
Alphabet:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Question 3. Graphics [15 marks]

(a) [5 marks] Show the output of the Edge List algorithm for the following picture. Show your working (i.e. intermediate or delta values) to ensure you will get all the credit. Round each coordinate to 1 decimal place as appropriate. The three vertices have the following coordinates:

- $V1 = (28, 10, 10)$
- $V2 = (24, 15, 20)$
- $V3 = (30, 12, 30)$



LEFT		RIGHT	
X	Z	X	Z

Your working:

(Question 3 continued on next page)

(Question 3 continued)

(b) [5 marks] Transform the point $p = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ into a point q with the following steps:

1. Translate by (2,2).
2. Scale by 3.
3. Rotate by 90 degrees (CCW).
4. Translate by (-1,1).
5. Rotate by 90 degrees (CCW).

Write an equation that expresses the relation between p and q. On the left hand side write the final coordinates of q (as a column vector), and on the right hand side write an algebraic expression of matrices, vectors, and p. Write explicitly the contents of each matrix and vector.

A reminder: $sin(90) = 1$, $cos(90) = 0$, and a 2D rotation matrix of α degrees can be expressed as $R = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$.

(Question 3 continued on next page)

(Question 3 continued)

(c) [5 marks] Write an equation that expresses the relation between p and q, but now in homogeneous coordinates. On the left hand side write q in homogeneous coordinates, and on the right hand side write a series of matrix multiplications by the point p. Write explicitly the contents of each matrix and vector in the equation (including q, in real numbers).

A reminder: Given a 2D rotation matrix R, its homegemous coordinates representation is $R_{3\times3} = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}$ (where each zero spans two columns or two rows).

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SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

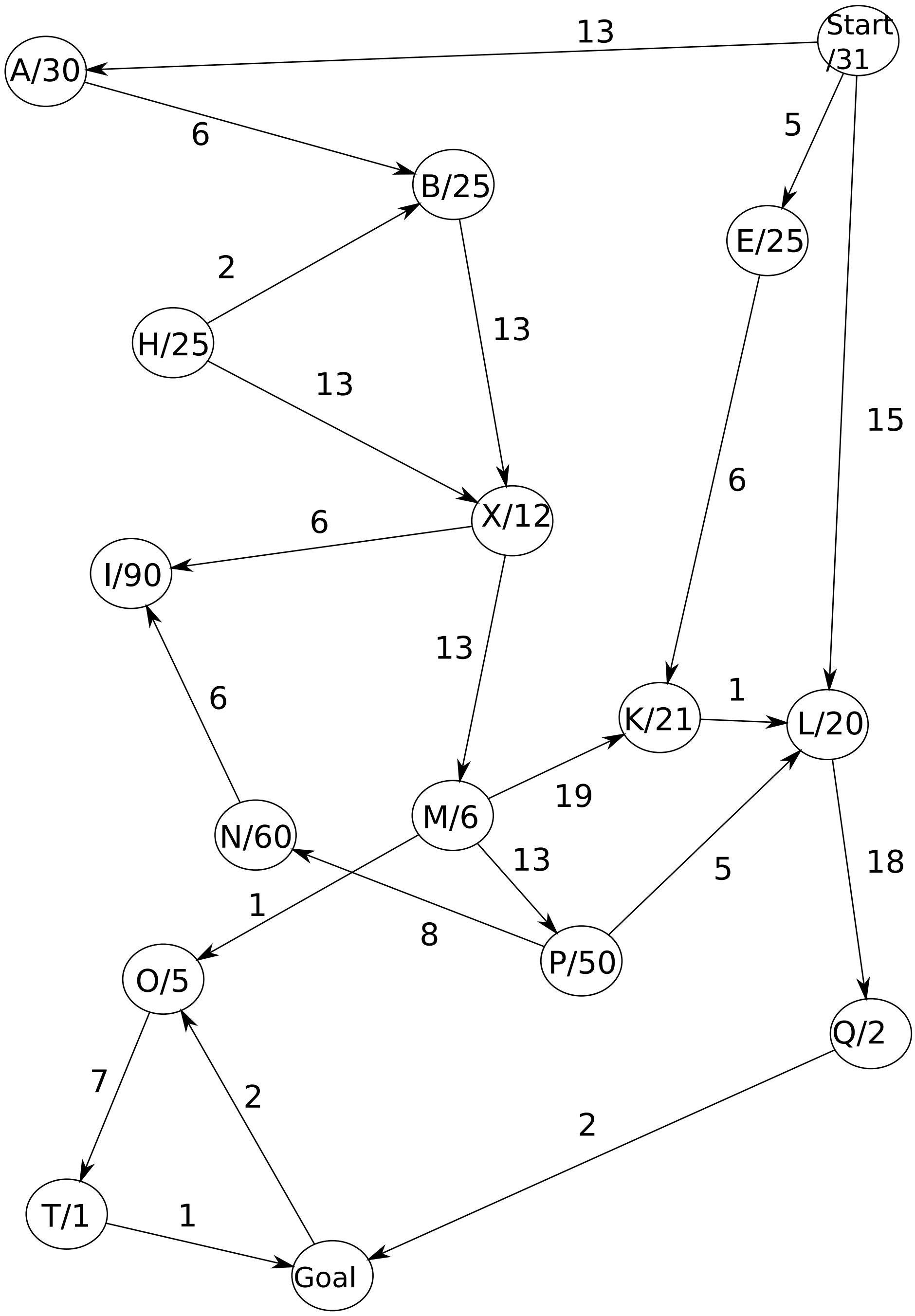

Figure 1: Graph for Question 1