Model Solutions
COMP 261 Test 4

27 May 2023

Instructions

- Time allowed: **50 minutes** .
- Answer **all** the questions. There are 50 marks in total.
- Write your answers in the booklets and remember to include your Student ID.
- If you are a remote student, then write the answers on paper and submit a photo to the relevant submission system entry.
- If you think some question is unclear, ask for clarification.
- This test contributes 25% of your final grade
- You may use paper translation dictionaries, and non-programmable calculators.
- You may write notes and working on this paper, but make sure your answers are clear.

Questions	**Marks**
1. Data Compression	[26]
2. String Search	[16]
3. Fast Fourier Transform	[8]

1. Data Compression **(26 marks)**

*****Huffman Coding*****

(a) **(8 marks)** The following text is to be encoded using Huffman coding, based on the letter frequencies in the text itself:

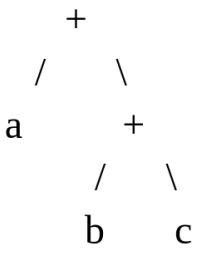
`to_be_or_not_to_be`

- Construct the relevant binary tree,
- Show the code it assigns to each character.

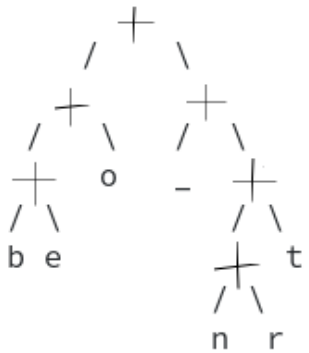
Note 1: If the two nodes have the same frequency in the priority queue, pick the node with the smaller character alphabetically. Specifically, "_" comes **before all the lowercase letters.*

Note 2: When building a parent node, use the child node with the smaller frequency as the **left child.*

**Note 3: You can use a "+" to represent the non-leaf nodes when drawing a Huffman tree, like:*



Draw your Huffman tree below:



The code for each character is:
t: 111
o: 01
_: 10
b: 000
e: 001
r: 1101
n: 1100

(b) **(2 marks)** Now encode the string using your above codebook and show the encoded binary bit stream:

111 01 10 000 001 10 01 1101 10 1100 01 111 10 111 01 10 000 001

(c) **(1 mark)** What is the compression rate compared with the result of a fixed-length coding method where 3 bits are used for representing one symbol?
Note: If your encoded sequence has 40 bits, but the fixed-length encoding give you 50bits, the compression rate is 40/50 = 80%

47 / 54 = 87%

(d) **(2 marks)** If we use the **above huffman tree** to encode the following sequence: `net_born_rent`
You will have a longer sequence than fixed-length coding. Please explain why using the previous Huffman coding tree can not result in a good compression rate for the new sequence?

Because the characters of the new string does not follow the same distribution of the characters in the original text. Different frequencies lead to different optimal Huffman trees.

EXTRA ANSWER BOX IF NEEDED (PLEASE INDICATE THE QUESTION IDs):

Lempel-Ziv Coding

(e) **(3 marks)** Suppose that the following tuple sequence is an encoding result by the Lempel-Ziv encoding method:

`[0, 0, 'b'][0, 0, 'e'][1, 1, 't'][0, 0, '_'][5, 3, '_'][9, 4, '']`

**Note: The final '' in the last tuple means an empty character*

Which one of the following four strings is the correct decoding result for the previous tuple sequence: [ ]

C

(A) beet_beet_bee

(B) bet_bet_bet_t

(C) beet_bee_beet

(D) beet_beet_bee

(f) **(4 marks)** For the following string:

`a_cat_catches_it`

Which option shows the correct encoding result using the Lempel-Ziv method? [ ]

B

Note: Here, we assume the length of the sliding search window behind the current character to be processed is 16 and the size of the "lookahead" window for the substring/pattern to search is 16.

(A) `[0,0,'a'][0,0,'_'][0,0,'c'][3,1,'t'][0,0,'t'][4,4,'h'][0,0,'h'][0,0,'e'][0,0,'s'][12,1,'i'][0,0,'i'][11,1,'']`

(B) `[0,0,'a'][0,0,'_'][0,0,'c'][3,1,'t'][4,4,'c'][0,0,'h'][0,0,'e'][0,0,'s'][12,1,'i'][11,1,'']`

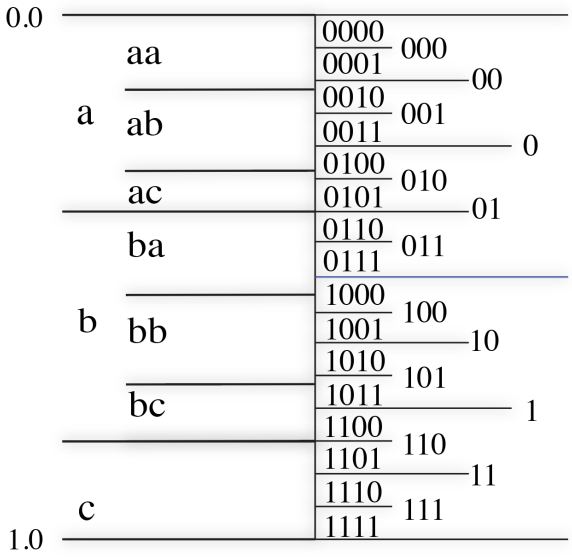
(C) `[1,1,'a'][1,1,'_'][1,1,'c'][3,1,'t'][4,4,'c'][1,1,'h'][1,1,'e'][1,1,'s'][12,1,'i'][11,1,'']`

(D) `[0,0,'a'][0,0,'_'][0,0,'c'][3,1,'t'][4,1,'_'][4,3,'c'][0,0,'h'][0,0,'e'][0,0,'s'][12,1,'i'][11,1,'']`

EXTRA SPACE TO WORK OUT THE QUESTIONS (WILL NOT BE MARKED)

Arithmetic Coding

(g) **(6 marks)** Suppose that we have an alphabet of {a, b, c}, with a probability distribution of {0.375, 0.4375, 0.1875}. Then we can have the following partitioning scheme:



Please choose the correct answer for each of the following questions:

(1) For the string "aa", what is the binary code sequence after encoding it using the arithmetic coding algorithm? [ ] A

(A) 000 (B) 0000 (C) 001 (D) 0

(2) For the string "ac", what is the binary code sequence after encoding it using the arithmetic coding algorithm? [ ] B

(A) 010 (B) 0101 (C) 0100 (D) 00

(3) When doing **on-the-fly** encoding for transmitting "aa", when read in the first 'a', what will be transmitted? [ ] D

(A) 1 (B) a (C) nothing (D) 0

(4) When doing **on-the-fly** encoding for transmitting "ba", when read in the first 'b', what will be transmitted? [ ] B

(A) b (B) nothing (C) 1 (D) 0

2. String Search **(16 marks)**

(a) **(6 marks)** Show the partial match table **M** generated for the KMP algorithm for the following search string **S**:
The string S: "ananxany"

S	a	n	a	n	x	a	n	y
M	-1	0	0	1	2	0	1	2

(b) **(10 marks)** Show the steps the KMP algorithm using the above partial match table of S will take when searching through the following text T until the match is found. You only need to show the **start position of each match attempt** and **the part that can been matched** before the fail position.

Pos	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
T	a	n	a	n	t	a	n	a	n	x	a	n	a	n	x	a	n	y

**Note: Please use the following format to show where the match attempts are for each step. One example has been given.*
Note:* ***You may not need to fill in all the following empty steps given in the box. Please just end if a result can be returned from the KMP algorithm.*

Match Attempt 1
Start Position in T: **T[0]** Partially Matched Characters: **"anan"**
Return a value? (If yes, please write down the value. Otherwise, write "No"): **No**

Match Attempt 2:
Start Position in T: T[2] Partially Matched Characters: "an"
Return a value? (If yes, please write down the value. Otherwise, write "No"):
No
Match Attempt 3:
Start Position in T: T[4] Partially Matched Characters: None
Return a value? (If yes, please write down the value. Otherwise, write "No"):
No
Match Attempt 4:
Start Position in T: T[5] Partially Matched Characters: "ananxan"
Return a value? (If yes, please write down the value. Otherwise, write "No"):
No
Match Attempt 5:
Start Position in T: T[10] Partially Matched Characters: "ananxany"
Return a value? (If yes, please write down the value. Otherwise, write "No"):
10
Match Attempt 6:
Start Position in T: Partially Matched Characters:
Return a value? (If yes, please write down the value. Otherwise, write "No"):

Match Attempt 7:
Start Position in T: Partially Matched Characters:
Return a value? (If yes, please write down the value. Otherwise, write "No"):

3. Fast Fourier Transform **(8 marks)**

(a) **(4 marks)** Suppose we are using FFT to evaluate the following polynomial $P(x)$:
$P(x) = x^7 + 6x^6 + 3x^5 + 9x^4 + x^3 + 2x^2 + 7$
What are the two lower-degree polynomials (P_{even} and P_{odd}) we should evaluate during the subsequent recursion step following the splitting of the polynomial P(x): [ ] D
Hint: P_{odd} is **NOT** directly obtained by combining all the odd-degree individual terms.

(A) $P_{even}(x^2) = 6x^6 + 9x^4 + 2x^2$, $P_{odd}(x^2) = x^7 + 3x^5 + x^3 + 7$
(B) $P_{even}(x^2) = 6x^6 + 9x^4 + 2x^2$, $P_{odd}(x^2) = x^6 + 3x^4 + x^2$
(C) $P_{even}(x^2) = 6x^6 + 9x^4 + 2x^2 + 7$, $P_{odd}(x^2) = x^7 + 3x^5 + x^3$
(D) $P_{even}(x^2) = 6x^6 + 9x^4 + 2x^2 + 7$, $P_{odd}(x^2) = x^6 + 3x^4 + x^2$

(b) **(4 marks)** When using FFT algorithm for polynomial multiplication, what are the **fourth roots of unity** that can be used as the four points to evaluate the values of an input polynomial of **degree 3**? Please choose from the following options: [ ] C

(A) $1, -1, 1+i, 1-i$
(B) $1, -1, i, 0$
(C) $1, -1, i, -i$
(D) $i, i+1, i+2, i+3$

**********END OF TEST 4 **********

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