Victoria University of Wellington

DEGREE EXAMINATIONS — 1997

(MID-YEAR)

COMP 303

Design and Analysis of Algorithms

Time Allowed: 3 hours

- 6 questions totalling 180 marks
- Answer all questions
- Assign approximately 1 minute per mark
- Answer all questions in the answer book provided
- No written materials, except authorised translation dictionaries, are permitted
- Calculators are not permitted

1. (a) Explain in plain English the meaning and usage of $\Omega()$, $O()$, $\Theta()$ and $o()$ in the analysis of algorithms.

[16 marks]

(b) Give justification for the *principle of invariance* which states that two different implementations of the same algorithm will not differ in efficiency by more than a multiplicative constant factor.

[10 marks]

2. (a) State the *principle of optimality.*

[5 marks]

(b) Explain the general structure of a greedy algorithm.

[10 marks]

(c) The simple, non-optimal sorting algorithm *Insertion-Sort* is given below. In what ways may Insertion-Sort be classified as a greedy algorithm? Do you agree with such a classification? Explain your answer (you might like to refer to your answer to the previous question).

[15 marks]

```
Algorithm Insertion-Sort(Array A)
BEGIN
   for i = 1 to length(A) do
      temp = a[i];
      loc = i;
      while a[loc−1] > temp do
         a[loc] = a[loc−1];
         loc = loc−1;
      a[loc] = temp;
END
```

3. Discuss the applicability of the Divide-and-Conquer, Dynamic Programming and Greedy Methods to the *Fractional Knapsack* and *0-1 Knapsack* problems.

[30 marks]

4. For some problems, a brute-force search of the solution space is the most efficient (known) algorithm. With reference to the n-queens problem, suggest some general mechanisms which may be used to limit the size of such a search space, and thereby improve the performance of a brute-force search in the average case.

[20 marks]

5. (a) Explain the *capacity constraint* and the notion of *flow conservation* as they apply to flow networks.

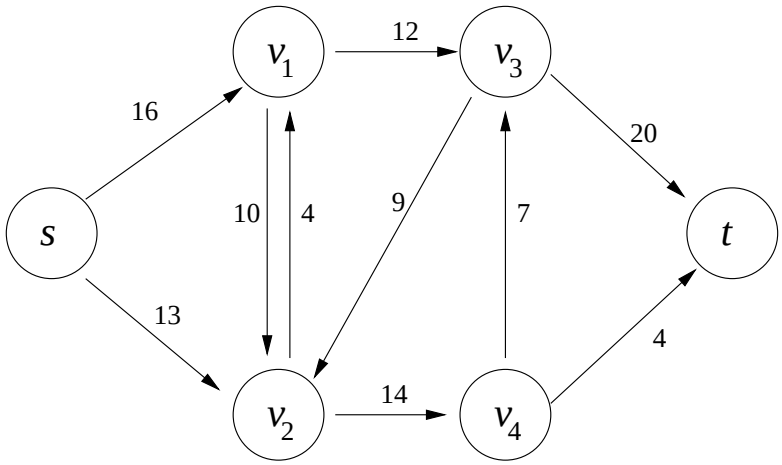
[5 marks]

(b) The Ford-Fulkerson and Edmonds-Karp algorithms for maximal flow make use of the concept of *residual networks*. What is a residual network?

[3 marks]

(c) For the following flow network, use the Ford-Fulkerson method to determine the value of the maximal flow. Be sure to show your working.

[14 marks]



(d) Explain the relationship between the minimum capacity cut and the maximal flow in an arbitrary flow network $G = (V, E)$, with source s and sink t.

[10 marks]

6. (a) Give definitions for the classes P, NP, *NP-complete* and *NP-hard*.

[12 marks]

(b) Prove that problem *0-1KNAPD*, given below, is a member of NP.

[10 marks]

Problem **0-1KNAPD**: *Given a set of n items $I = (i_1, i_2, i_3, ..., i_n)$ having weights $w_1, w_2, w_3, ..., w_n$ kg and dollar values $v_1, v_2, v_3, ..., v_n$ respectively, does there exist a set of items $B \subseteq I$ such that the total weight of B is less than or equal to some capacity W kg, and whose total value is greater than k dollars?*

(c) One of the fundamental questions in computer science today is the so-called $P = NP$ problem. Discuss the meaning of this problem, and provide a conjecture as to its solution. Provide supporting evidence for your conjecture.

[20 marks]

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