TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

Student ID: .....

EXAMINATIONS — 2012

TEST

SWEN 224
Formal Foundations of Programming

Time Allowed: 50 minutes

Instructions:
- There are 50 possible marks.
- Answer all questions in the boxes provided.
- If additional space is required you may use one of the spare pages, and indicate where your answer is in the box for that question.
- Write clearly and cross out rough working and anything else you don't want marked.
- Non-electronic foreign language dictionaries are allowed.
- Calculators ARE NOT ALLOWED (and not required).
- No other reference material is allowed.

Question	Topic	Marks	Achieved
1.	Understanding Assertions	12	
1.	Writing Assertions	12	
2.	Verification	12	
3.	Loops	14	
	Total	50	

Student ID: .....

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Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Student ID: .....

Question 1. Understanding Assertions [12 marks]

Consider the following assertion:

$$\exists\, i, j : 1..\,|A| \bullet A[j] = A[i] + A[i+1]$$

where A is assumed to be an array on numbers, with indexes starting from 1.

(a) [8 marks] For each of the following arrays, say whether the assertion is true or false for that array:

(i) $A = (1, 2, 3)$

(ii) $A = (5, 3, 15, 6)$

(iii) $A = (0, 0)$

(iv) $A = (0)$

(b) [4 marks] State in words what the assertion means

Student ID: .....

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Student ID: .....

Question 2. Writing Asertions [12 marks]

Write an assertion mathematical/logical expression to formalise each of the following statements.

(a) [3 marks]

The values of x and y are both greater than w and both less than or equal to z.

(b) [3 marks] The values in array A are in srictly descending order.

(c) [3 marks] Some value in array A occurs only once.

(d) [3 marks] Array B is a *rotation* of array A, where rotating an array means taking some number, say k between 0 and $|A| - 1$, of elements from the end of the array, moving the rest of the array up k places, and placing the removed elements at the from of the array. For example, $(1,2,3,4)$, $(4,1,2,3)$, $(3,4,1,2)$, $(2,3,4,1)$ are all reotations of array $(1,2,3,4)$.

Student ID: .....

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Question 3. Verifying Simple Programs [12 marks]

For each of the following corrctness assertion, give the verification condition required to demonstrate that the correctness assertion is valid.

You are not required to prove these verification conditions.

(a) [3 marks] $\{x \neq 0 \wedge y \neq 0\}\, x := x + y\, \{x \neq 0\}$

(b) [6 marks]

$\{1 \leq k < |A| \wedge s = \sum_{i=1}^{k} A[i]\}$ **if** $A[k+1] \neq 0$ **then** $s := s + A[k+1]$ **fi** $\{s = \sum_{i=1}^{k+1} A[i]\}$

(c) [3 marks] $\{A[1] = 5 \wedge i = 1\}\, A[i] := A[i] + A[i]\, \{A[1] = 10\}$

Question 4. Loops [14 marks]

(a) [6 marks] The following loop sums the elements of an array A (indexed from 1), along with pre and postconditions shown as assertions:

assert $k = 0 \wedge s = 0$
while $k < |A|$ **do**
$k := k + 1$;
$s := s + A[k]$
od;
assert $s = \sum_{i=1}^{|A|} A[i]$

The loop invariant is $0 \leq k \leq |A| \wedge s = \sum_{i=0}^{k} A[i]$.

State the verification conditions that must be proved in order to verify the loop.

You are not required to prove these verification conditions.

(b) The following are three lternative ways of writing the algorithm given above.

In each case, give a loop invariant that could be used to veriy the loop, and state any additional preconditions that may be required.

(i) [2 marks]

assert $k = |A| \wedge s = 0$
while $k > 0$ **do**
$s := s + A[k]$;
$k := k - 1$
od;
assert $s = \sum_{i=1}^{|A|} A[i]$

(ii) [3 marks]

assert $k = 2 \wedge s = A[1]$
while $k \leq |A|$ **do**
$k := k + 1$;
$s := s + A[k]$
od;
assert $s = \sum_{i=1}^{|A|} A[i]$

(iii) [3 marks]

assert $k = 1 \wedge s = 0$
while $k \leq |A|$ **do**
$s := s + A[k] + A[k+1]$;
$k := k + 2$
od;
assert $s = \sum_{i=1}^{|A|} A[i]$

Student ID: .....

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