

# Tribal Ownership

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## Abstract

*Tribal Ownership* unifies class nesting and object ownership. Tribal Ownership is based on Tribe, a language with nested classes and object families. In Tribal Ownership, a program’s runtime object ownership structure is characterised by the lexical nesting structure of its classes.

We build on a variant of Tribe to present a *descriptive* ownership system, using object nesting to describe heap partitions, but without imposing any restrictions on programming disciplines. We then demonstrate how a range of different *prescriptive ownership policies* can be supported on top of the descriptive Tribal Ownership mechanism; including a novel *owners-as-local-dominators* policy. We formalise our type system and prove soundness and several ownership invariants. The resulting system requires strikingly few annotations, and uses well-understood encapsulation techniques to create ownership systems that should be intuitive for programmers.

**Categories and Subject Descriptors** D.3.3 [Software]: Programming Languages—Language Constructs and Features

**General Terms** Languages, Theory

**Keywords** Ownership types, virtual classes, nested classes, family polymorphism

## 1. Introduction

**Ownership Types** Ownership type systems statically impose hierarchical structures onto the heap. By structuring the heap, compilers and programmers can reason about small parts of it in isolation, which makes many otherwise intractable analyses possible. Ownership languages often support encapsulation policies based on the heap structure (*prescriptive* systems); these might restrict references, modification, or access. Alternatively, ownership information can be

used to describe the effects of computation without imposing any restrictions (*descriptive* ownership) [16]. Ownership types have been used to support memory management and garbage collection [10, 51], alias analysis [2], verification [25], concurrency [7, 24, 50], parallelisation [21, 18], real-time programming [46, 51], and more [2, 23, 9].

Ownership types tend to require substantial annotations to describe how individual fields and methods relate to the ownership hierarchy. These annotations can be confusing, and have negative side-effects on refactoring and program maintenance. Furthermore, because the annotations primarily decorate individual methods and fields, it can be difficult to see how each class or object fits into the ownership structure as a whole.

In this paper we present an ownership system based on virtual classes that does not require any additional annotation, and, by making the ownership structure explicit in a program’s class structure, should be easier for programmers to use and understand.

**Virtual Classes** Virtual classes allow *families* of classes to be inherited, rather than just single classes. In a language with virtual classes, classes are lexically nested inside other classes. When a class is inherited, its nested inner classes (and the relationships between them) are inherited along with its methods and fields. Virtual classes allow programs to express types such as “all nodes of any graph”, “all nodes belonging to the graph  $g$ ”, and “this particular node in this particular graph”, and to distinguish between them statically.

Virtual classes are a powerful and intuitive mechanism which are beginning to appear in practical programming languages [36, 12]. We expect virtual classes to become more mainstream; our proposal uses virtual classes to bring the considerable benefits of ownership types to a language at low additional cost.

**Our Contribution** In this paper we describe *Tribal ownership*, an ownership types system built on a variant of the virtual class calculus Tribe [19]. Our language has no additional syntax to support ownership, only the usual syntax used to support virtual classes. In Tribal Ownership, the object ownership and nested class hierarchies coincide. By unifying these hierarchies we make the ownership hierarchy

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more apparent in source code and the system easier for programmers to understand.

In our proposed system, all instances of a class are owned by the *object* which encloses that class. Since this object is already tracked by the type system, no additional annotations are required to enforce ownership. Flexibility is provided by allowing classes to be *imported* into a different class, and so take instances of the importing class as owners.

We show how different encapsulation policies can be enforced on top of the descriptive Tribal Ownership system. Additionally, we present a new variation on the standard owners-as-dominators policy that enforces owners-as-dominators locally within isolated sub-heaps, whilst giving full access to other areas of the heap.

In summary, the contributions of this paper are:

- A new variant of the Tribe language with simpler types, cross-family inheritance, and generics. We believe this variant is easier to program with and understand than the original version of Tribe. We also contribute a new formalism (in which, formalising cross-family inheritance was the main challenge) and type soundness proof.
- The first formalisation of ownership types using virtual classes (Tribe<sub>own</sub>), and a proof that the ownership hierarchy is preserved by our type system.
- Formal and informal descriptions of several different encapsulation policies built on top of Tribal ownership.
- The novel encapsulation policy owners-as-*local*-dominators, a flexible policy which subsumes owners-as-dominators. We prove that this property is enforced by our formalism.

**Organisation** We give background on the Tribe language in Sect. 2. We describe our new variant, Tribe<sub>own</sub>, its formalisation, and soundness proof in Sect. 3. We describe how this system supports descriptive ownership, and extend our formalism in order to prove sound ownership typing in Sect. 4. We describe how encapsulation policies can be encoded in Tribe<sub>own</sub> and describe and formalise our new encapsulation property owners-as-local-dominators in Sect. 5. We cover related work in Sect. 6, and further work and our conclusions in Sect. 7.

## 2. Tribe

Tribe (along with many other languages following on from BETA [37]) supports *nested virtual classes*. As with Java’s (non-static) inner classes, Tribe’s nested class declarations lead to nested objects at runtime. Consider the following simple example, inspired by [40]:

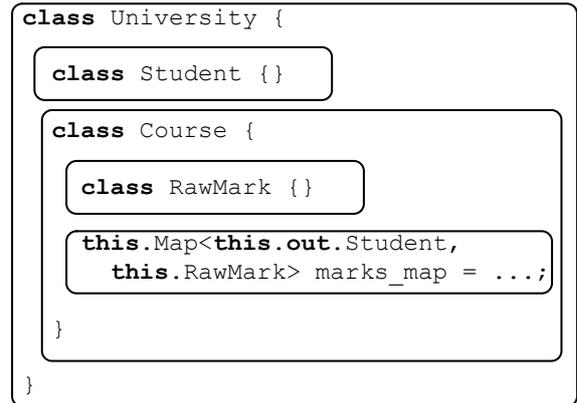


Figure 1. Nesting structure of nested classes

```

class University {
  class Student {...}
  class Course {
    class RawMark {...}
    this.Map<this.out.Student,
      this.RawMark> marks_map = ...;
  }
}

```

This code defines a University class with two nested classes inside it: one class for modelling Students, and another for modelling Courses. Each Course object has a RawMark class, representing the private, unmoderated marks for each course, and a Map which relates students to their raw marks. Figure 1 shows how the lexical structure of the class definitions imposes a nesting relationship on the classes.

The marks\_map field maps students (**this.out.Student**, i.e., student objects from the enclosing university object) to raw marks (**this.RawMark**). The respective types reflect that Student is defined one level out in the nesting hierarchy, and RawMark is defined inside the current class (Course).

### 2.1 Nested Objects

Fig. 2 shows an example of how these nested classes could be instantiated at runtime. The outermost University class is instantiated twice, with one instance representing “VUW” and the other representing “Uppsala”. Nested inside each university *object* are a number of different objects, instances of the Student and Course classes. Just as the Student and Course class declarations are nested inside the declaration of the University class in the program source code, so the various Student and Course instances are nested inside the University object to which belong. Going further down inside the VUW course SWEN221, inside that course object are Map and three RawMark objects. Being nested inside another object has no effect on references between objects: the Map refers to those three RawMarks and also to three students who

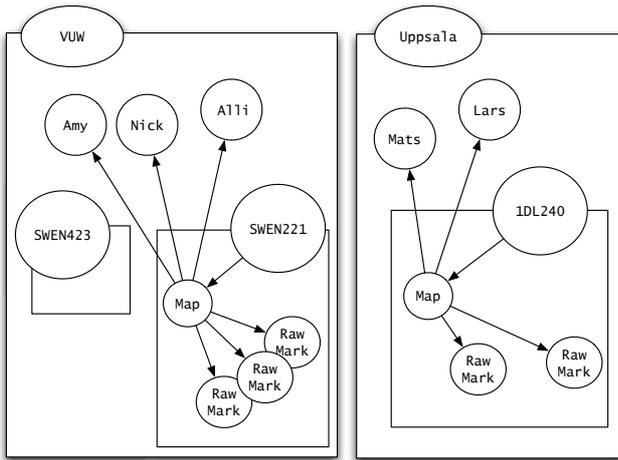


Figure 2. Objects instantiating nested classes

are presumably taking those courses (and have the associated raw marks) but who are outside the Course object.

In Tribe, inner class instances maintain a runtime reference to their enclosing object: this is written “out”. So **out** for Mats, Lars, and 1DL240 is the Uppsala object; while **out** for Amy, Nick, Alli, SWEN423, and SWEN221 is the VUW object. This is illustrated in Fig. 3. Note the ? on the dotted line from Map to Uppsala, which we will return to when discussing cross-family imports later.

The **out** above is the same **out** used in defining types: conceptually, the *types* of the inner classes are also nested within their enclosing object. Every object anchors a *class family*: objects in the class are only compatible (related by subtyping) with objects in the same family. So the courses and students in each university are *different* types from *different* families: a VUW student cannot enroll at an Uppsala course, and vice versa [29, 19].

## 2.2 Family Polymorphism

Tribe supports a powerful notion of inheritance which allows groups of classes to be extended together, preserving the relationships between classes. Nested classes can be inherited or overridden along with a superclass’s fields and methods. This is known as *family polymorphism* [29], and supports sophisticated implementations for design patterns and the expression problem [48, 47]. The following example shows how a Conservatorium — a stand-alone music school — can be implemented by inheriting from a University:

```
class Conservatorium extends University {
  class Student {this.out.Instrument major; ...}
}

class Instrument {...}
```

Because they include nested inner classes, University and Conservatorium define two class *families*, which are related by inheritance. So, Conservatorium will not only inherit

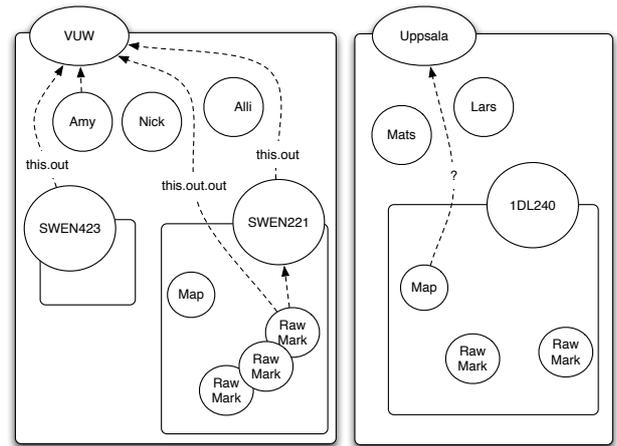


Figure 3. Paths from nested to enclosing objects.

methods and fields from University, but also University’s Course and Student classes. Because every conservatorium student must major in a musical instrument, Conservatorium.Student *further binds* University.Student: adding in a major field of type Instrument. This is called further binding because by class family inheritance, a Conservatorium.Student (i.e. the Student class declared within Conservatorium) will also gain all the fields and methods from the University. Student student — and any inner classes too!

Subtyping in Tribe follows from subclassing, but not further binding. For example, Conservatorium is a subtype of University; but Conservatorium.Student is not a subtype of University.Student

## 2.3 Tribe Types

Tribe uses *path dependent types* to distinguish classes which belong to different families. For example, assuming the variables University *u* and Conservatorium *c*, the type *c.Course* is not a subtype of *u.Course*, because *u* and *c* might refer to different objects. *Object family types* consist of a *path* to an object and the name of a class defined by that object. Paths start at a variable (often **this**) and have steps using final fields or the **out** keyword (which denotes a class’s enclosing object). For example, in an object with type *u.Course*, the path **this.out**, will refer to a University object, in an object of type *c.Course*, the same path will refer to a Conservatorium object.

Tribe types form a spectrum: from *singleton types* which consist of a path only and denote a specific object; object family types which represent instantiations of a class nested in a specific object; to *absolute types* which specify only the class. In our running example, *u* is a singleton type denoting the specific university object referred by the variable; *u.Course* is an object family type denoting any course in *u*, and the absolute type *University.Course* is any course in any university.

Subtyping only occurs within an object family: `u.Course` is a subtype of `u.Course` (by reflexivity), but `u1.Course` is not a subtype of `u2.Course`; all three types are subtypes of their corresponding absolute type, `University.Course` (assuming `u`, `u1`, `u2` are instances of `University`).

Tribe is not only expressive, but *intuitive*: nested classes are easy to understand, families of classes are a natural unit of code, and paths are familiar from Java programming.

### 3. Tribe<sub>own</sub>

Our goal has been to support ownership types in a virtual classes language. We chose Tribe as our base language because of its flexible types, elegant and fairly standard formalism, and because the type system is amenable to capturing ownership information. Supporting object ownership in Tribe is not trivial, however: we had to improve support for cross-family inheritance (described below in Sect. 3.2), change some aspects of the formalisation (Sect. 3.3) and add ways to reason about and enforce ownership properties (Sect. 4 and Sect. 5). We call our variant language Tribe<sub>own</sub>.

In Tribe<sub>own</sub>, we take the hierarchy of nested classes as the ownership hierarchy. An object’s owner is its enclosing object. For any type `T`, `T.out` (the type of the enclosing object) is the owner of objects with type `T`.

#### 3.1 Path Types and generics

Tribe<sub>own</sub> has a more restrictive syntax of types than Tribe: we disallow the use of final field names in types and restrict the ordering of the components of types, in effect, imposing a normal form for types. This is necessary to conceptualise and prove some ownership properties (discussed in Sect. 4). It also simplifies our formalisation.

Tribe types have syntax `x.(ff | out | C)*`, where `x` includes `this` and the outermost ‘world’ object, `world`, `ff` is the name of a final field, and `C` is a class name. Tribe<sub>own</sub> types have the form `x.out*.C*`; that is, a path which starts at a variable, then goes outward some number of steps, and then inward by some number of class names. We support absolute types (where the path starts at `world` and has no outward steps), singleton types (where there are no class names), and all types with an intermediate level of precision (e.g., `this.out.C.D`). We do not support types with fields<sup>1</sup> or types which have an outward step from a class name. Types of the latter case can be represented in Tribe<sub>own</sub> by using the equivalent type, which has neither class name nor outward step, e.g., `x.D` for `x.C.out.D`.

To make up some of the expressivity lost by removing fields from types, we introduce simple type genericity. For now, we only support unbounded generics on classes and

<sup>1</sup>We found field types uncommon in our experience, and often restrictive because the Tribe type system cannot associate fields used in types with the values instantiating the fields; e.g., even if `x.f = y`, there is no relationship between `y.C` and `x.f.C`.

types. Generic methods should be easy to support and are elided. Bounded and variant generics are left for future work.

#### 3.2 Importing Module Classes

Tribe<sub>own</sub> supports cross-family inheritance through *module classes*. Module classes restrict external dependencies by forbidding the use of `out` in their definitions. Since module classes do not have any external dependencies, they (and their nested classes) can be imported arbitrarily. As the code example below shows, `Map` is a module class, and therefore cannot name any external object. Hence, the dotted-line from `Map` to `Uppsala` in Fig. 3, cannot exist.

Module classes may only extend other module classes. We elaborate the example from Sect. 2:

```

module Collections {
  class List<X> {...}
  class Map<K, V> {...}
}

class University {
  ...
  class Course imports Collections{
    ...
    this.Map<this.out.Student,
      this.RawMark> marks_map = ...;
  }
}

```

Importing a module gives subtyping: in the example above, `Course` is a subtype of `Collections` and `Course.List<University>` is a subtype of `Collections.List<University>`. Module classes are reminiscent of mixin modules, which lack a super class, except that the dependency is on nesting rather than on subclassing.

In our formalism (described below), `out` is modelled as a distinguished field. A module class is simply a class which does not define an `out` field. Thus, we check whether an `out` field is defined or not to check if a class is a module, and can thus be imported.

Cross-family inheritance via module classes allows Tribe<sub>own</sub> to have a single top type. Tribe lacks a top type — each class has a nested `Object` class which is unrelated to other `Object` classes. We assume a module class `Root` nested directly inside `world` which contains an `Object` class. All classes (including `world`) implicitly import `Root`, and all classes without explicit superclasses are implicit subclasses of the adopted `Object` (a ‘local’ top type). Every local `Object` is a subtype of `Root.Object`, which is thus a top type for Tribe<sub>own</sub>. We can, therefore, describe any object nested within an object or class using a local `Object` (e.g., `x.Object` is the superclass of all classes nested in `x`) and describe all objects using `Root.Object`. A top type is useful for describing variables to be used as owners and, hypothetically, for bounds on type variables.

Tribe did not support cross-family polymorphism, although an *adoption* mechanism was proposed (but not

proved sound) which addressed some of the same motivations. Module classes are a more developed solution: we formalise their interaction with subclassing and subtyping, and include them in our soundness proof.

### 3.3 Formalisation

Our formalisation of  $\text{Tribe}_{\text{own}}$  borrows heavily from the Tribe formalism. Differences reflect differences in the language design, some simplifications, a focus on ownership rather than general language design, and differences in style. Our formalism is in the Featherweight Java [32] mould.

Languages with multiple inheritance (including  $\text{Tribe}_{\text{own}}$  and Tribe) must deal with the possibility of ambiguous lookup, where fields and methods with the same name are inherited from several different superclasses. Tribe sidesteps this problem by non-deterministically selecting one. This is not an issue for soundness because all methods inherited by a class with the same name, must have the same type. We go a step further and do not even guarantee that a method from the most specific class will be executed, only that some inherited method will be executed. Although this decision would be impractical in a real language, it does not affect our soundness result because our (non-deterministic) calculus is more general than a corresponding language with deterministic dispatch. In a practical implementation, we can select from a variety of solutions, e.g., forcing the programmer to manually resolve ambiguities.

**Syntax** The syntax of  $\text{Tribe}_{\text{own}}$  is defined in Fig. 4. Elements in grey may appear during evaluation, but may not be written by a programmer. The syntax of expressions will be described when we cover their type rules. The key feature of class declarations is that they may include nested classes. We use  $\triangleleft$  for “**extends**” and  $\nabla$  for “**imports**”. The former indicates subclassing, the latter importing a module. We discuss public and private classes in Sect. 5. We treat **this** as a distinguished variable and **out** as a distinguished field.

Environments ( $\Gamma$ ) can contain mappings from variables to their types, type variables (showing which type variables are in scope), and a heap ( $\mathcal{H}^\Gamma$ , used as an environment for type checking runtime expressions).

Our rules make use of substitution. Because of the flexibility of types in Tribe, any type can be substituted for a variable in a type and give a syntactically valid type. This is not the case in  $\text{Tribe}_{\text{own}}$ ; for example,  $[y.C/x]^{std}x.out.D$  gives  $y.C.out.D$  which is not a  $\text{Tribe}_{\text{own}}$  type (it is valid in Tribe). To address this, we define substitution in a non-standard way in Fig. 5. In this figure we use  $[...]^{std}$  to mean standard substitution.  $\text{Tribe}_{\text{own}}$  substitution is thus defined to normalise ill-formed types into  $\text{Tribe}_{\text{own}}$  types by matching and eliminating **outs** with class names. For example,  $[y.C/x]x.out.D$  is defined to give  $y.D$  in  $\text{Tribe}_{\text{own}}$ , and  $[y.out.C/x]x.out.D$  to give  $y.out.D$ .

**Inheritance**  $\text{Tribe}_{\text{own}}$  has two forms of inheritance: subclassing and further binding. These relations (between ab-

$e$	$::=$	$\text{null} \mid p \mid \gamma.f \mid \gamma.f = e$	<i>expressions</i>
		$\mid \text{let } x:T = e \text{ in } e \mid \gamma.m(\bar{\gamma})$	
		$\mid \text{new } \gamma.N \mid \text{err}$	
$v$	$::=$	$\text{null} \mid \text{world} \mid \iota$	<i>values</i>
$\mathcal{P}$	$::=$	$\bar{Q} e$	<i>programs</i>
$Q$	$::=$	$\alpha \text{ class } C \triangleleft \bar{X} \triangleleft \bar{N} \nabla \bar{A} \{ \bar{Q} \bar{T} \bar{f}; \bar{M} \}$	<i>class declarations</i>
$\alpha$	$::=$	$\text{public} \mid \text{private}$	<i>naming modifiers</i>
$M$	$::=$	$T m(\bar{T} \bar{x}) \{ \text{return } e; \}$	<i>method declarations</i>
$s$	$::=$	$\gamma \mid \text{world}$	<i>path starts</i>
$p$	$::=$	$s \mid p.out$	<i>paths</i>
$N$	$::=$	$C \triangleleft \bar{T}$	<i>instantiated class names</i>
$P$	$::=$	$p \mid P.N$	<i>path types</i>
$T, U$	$::=$	$P \mid X$	<i>types</i>
$A$	$::=$	$\text{world} \mid A.N$	<i>absolute types</i>
$r$	$::=$	$\text{world} \mid \iota$	<i>runtime paths</i>
$R$	$::=$	$r.C \triangleleft \bar{R}$	<i>runtime types</i>
$\Gamma$	$::=$	$\emptyset \mid \Gamma, \gamma:T \mid \Gamma, X \mid \mathcal{H}^\Gamma$	<i>environments</i>
$\gamma$	$::=$	$x \mid \iota$	<i>variables or addresses</i>
$\mathcal{H}$	$::=$	$\emptyset \mid \mathcal{H}, \iota \rightarrow \{R; \bar{f} \rightarrow v\}$	<i>heaps</i>
$x, \text{this}$			<i>variables</i>
$X$			<i>type variables</i>
$C, D, \text{Object}$			<i>class names</i>
$f, out$			<i>fields</i>

Figure 4. Syntax of  $\text{Tribe}_{\text{own}}$ .

$$\frac{[T/\gamma]^{std}T' = p.\bar{C}.out.\bar{C}' \quad |\bar{C}| = |\bar{out}|}{[T/\gamma]T' = p.\bar{C}'}$$

Figure 5. Type substitution in  $\text{Tribe}_{\text{own}}$ .

solute types) are defined in Fig. 6. The auxiliary function  $\mathcal{P}$  (defined in Fig. 9) looks up the definition of a class in the program, taking into account type substitution. Sub-classing reflects the relationship between classes denoted by the programmer using the **extends** keyword (SC-SUBCLASS). When a class is inherited, the subclass relationships between its child classes are also inherited (SC-NEST). (SC-IMPORT) accounts for subclassing due to imported modules. Further binding is the implicit inheritance of child classes when a surrounding class is inherited (FB-NEST). For concreteness,  $B.C$  further binds  $A.C$  and subclasses  $B.D$  in the following:

```

class A {
  class C {}
}
class B extends A {
  class D {}
  class C extends D {}
}

```

---

**Subclassing**  $\boxed{\vdash A \sqsubset_s A}$

$$\frac{\alpha \text{ class } C \langle \bar{X} \rangle \triangleleft \bar{N} \dots \in \mathcal{P}(A)}{\vdash A.C \langle \bar{T} \rangle \sqsubset_s A.[\bar{T}/\bar{X}]N_i}$$

(SC-SUBCLASS)

$$\frac{\mathcal{P}(A) \text{ defined} \quad \vdash A \sqsubset_i A' \quad \vdash A'.N \sqsubset_s A'.N'}{\vdash A.N \sqsubset_s A.N'}$$

(SC-NEST)

$$\frac{\alpha \text{ class } C \langle \bar{X} \rangle \dots \nabla \bar{A} \dots \in \mathcal{P}(A)}{\vdash A.C \langle \bar{T} \rangle \sqsubset_s [\bar{T}/\bar{X}]A_i}$$

(SC-IMPORT)

**Further Binding**  $\boxed{\vdash A \sqsubset_f A}$

$$\frac{\vdash A \sqsubset_i A' \quad \alpha \text{ class } C \langle \bar{X} \rangle \dots \in \mathcal{P}(A')}{\vdash A.C \langle \bar{T} \rangle \sqsubset_f A'.C \langle \bar{T} \rangle}$$

(FB-NEST)

**Inheritance**  $\boxed{\vdash A \sqsubset_i A}$

$$\frac{\vdash A \sqsubset_s A'}{\vdash A \sqsubset_i A'} \quad \frac{\vdash A \sqsubset_f A'}{\vdash A \sqsubset_i A'}$$

(I-SC) (I-FB)

---

**Figure 6.**  $\text{Tribe}_{own}$  inheritance and subclassing.

**Well-formed and absolute types** An absolute type ( $A$ ) describes an object by a path from the **world** program root, down through the class tree, for example, **world.C.D**. The judgement  $\Gamma \vdash P \uparrow A$  holds if  $A$  is an absolute type for all objects of type  $P$ . A non-variable type in  $\text{Tribe}_{own}$  is well-formed if a corresponding absolute type can be derived. Absolute types and well-formed types are defined in Fig. 8<sup>2</sup>. The rules that define absolute types are a combination of standard well-formedness rules for languages such as FGJ [32], and an inductive navigation of the nested class hierarchy<sup>3</sup>.

Well-formed heaps are defined in the same figure. A heap is well-formed if each object in it has a well-formed runtime type, a sensible **out** field, and all other fields are well-typed.

**Subtyping** Subtyping rules are given in Fig. 7. Subtyping follows from subclassing and importing ((S-SUBCLASS) and (S-IMPORT)), and the expected semantics of **out** (S-OUT-2).

<sup>2</sup>The  $fType$  auxiliary function looks up the type of a field and is defined in Fig. 9.

<sup>3</sup>Compared to Tribe, we require an inheritance premise in (A-CLASS) (as well as (SC-NEST)) because we do not have a  $cls$  judgement which takes inheritance into account.

---

**Subtyping**  $\boxed{\Gamma \vdash T <: T}$

$$\frac{}{\Gamma \vdash T <: T}$$

(S-REFLEX)

$$\frac{\Gamma \vdash T_1 <: T_3 \quad \Gamma \vdash T_3 <: T_2}{\Gamma \vdash T_1 <: T_2}$$

(S-TRANS)

$$\frac{\Gamma \vdash P \uparrow A \quad \vdash A.N \sqsubset_s A.N'}{\Gamma \vdash P.N <: P.N'}$$

(S-SUBCLASS)

$$\frac{\Gamma \vdash P <: P' \quad \Gamma \vdash P' <: P'}{\Gamma \vdash P <: P'}$$

(S-NEST-CLASS)

$$\frac{\Gamma(\gamma) = P}{\Gamma \vdash \gamma <: P}$$

(S-VAR)

$$\frac{\Gamma \vdash p \uparrow A.N \quad fType(\mathbf{out}, A.N) \text{ defined}}{\Gamma \vdash p <: p.\mathbf{out}.N}$$

(S-OUT-2)

$$\frac{\Gamma \vdash P <: P' \quad \Gamma \vdash P \uparrow A \quad \Gamma \vdash P' \uparrow A' \quad fType(\mathbf{out}, A') \text{ defined}}{\Gamma \vdash [P/x]x.\mathbf{out} <: [P'/x]x.\mathbf{out}}$$

(S-OUT-NEST)

$$\frac{\Gamma \vdash P \uparrow A' \quad \vdash A' \sqsubseteq_i^* A.C \langle \bar{T} \rangle \quad \alpha \text{ class } C \langle \bar{X} \rangle \dots \nabla \bar{A} \dots \in \mathcal{P}(A)}{\Gamma \vdash P <: [\bar{T}/\bar{X}]A_i}$$

(S-IMPORT)

---

**Figure 7.**  $\text{Tribe}_{own}$  subtyping.

Variables may be used as types; subtyping reflects that a variable's type is a less accurate way to describe the variable than the variable itself (S-VAR).

Types denote sets of objects, and a subtype is a smaller set of objects; (S-NEST-CLASS) and (S-OUT-NEST) reflect that we can go up or down the nested class hierarchy and preserve this 'smaller set of objects' concept. (S-OUT-NEST) must use substitution rather than plain nesting (that is,  $[P/x]x.\mathbf{out}$  rather than  $P.\mathbf{out}$ ) to account for types which are semantically 'out' but do not use the **out** keyword. Furthermore, we take advantage of the definition of  $\text{Tribe}_{own}$  substitution. For example, we can derive that  $x.\mathbf{out}$  is a subtype of **world.C**, if we can derive that  $x$  is a subtype of **world.C.D**. By using (S-OUT-NEST) in this way, (S-VAR), and (S-NEST-CLASS) we can derive that any type is a subtype of its absolute type, and so the rule (S-ABS) of Tribe is admissible in  $\text{Tribe}_{own}$ .

**Type checking** Auxiliary functions are defined in Fig. 9. We define field type, and method type and body lookup functions and the lookup function *method*, which checks if a method is defined. These are all defined declaratively using the inheritance relation, as opposed to the usual inductive

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**Absolute types**  $\boxed{\Gamma \vdash P \uparrow A}$ 

$$\frac{\Gamma(\gamma) = P \quad \Gamma \vdash P \uparrow A}{\Gamma \vdash \gamma \uparrow A} \quad \text{(A-VAR)} \qquad \frac{}{\Gamma \vdash \mathbf{world} \uparrow \mathbf{world}} \quad \text{(A-WORLD)}$$

$$\frac{\Gamma \vdash p \uparrow A.N \quad fType(\mathbf{out}, A.N) \text{ defined}}{\Gamma \vdash p.\mathbf{out} \uparrow A} \quad \text{(A-OUT)}$$

$$\frac{\Gamma \vdash P \uparrow A \quad \vdash A \sqsubseteq_i^* A' \quad \alpha \text{ class } C\langle\bar{x}\rangle \dots \in \mathcal{P}(A') \quad |\bar{T}| = |\bar{x}| \quad \Gamma \vdash \bar{T} \text{ OK}}{\Gamma \vdash P.C\langle\bar{T}\rangle \uparrow A.C\langle\bar{T}\rangle} \quad \text{(A-CLASS)}$$

**Well-formed types**  $\boxed{\Gamma \vdash T \text{ OK}}$ 

$$\frac{\Gamma \vdash P \uparrow A}{\Gamma \vdash P \text{ OK}} \quad \text{(F-PATH-TYPE)} \qquad \frac{x \in \Gamma}{\Gamma \vdash x \text{ OK}} \quad \text{(F-TYPE-VAR)}$$

**Well-formed heaps**  $\boxed{\vdash \mathcal{H} \text{ OK}}$ 

$$\frac{\forall \iota \rightarrow \{\iota'.C\langle\bar{R}\rangle; \bar{f} \rightarrow v\} \in \mathcal{H} \quad \bar{f}_i = \mathbf{out} \Rightarrow v_i = \iota' \quad \mathcal{H} \vdash \iota'.C\langle\bar{R}\rangle \uparrow A \quad fType(\bar{f}, A) = \bar{T} \quad \mathcal{H} \vdash v : [\iota/\mathbf{this}]\bar{T}}{\vdash \mathcal{H} \text{ OK}} \quad \text{(F-HEAP)}$$

**Figure 8.**  $\text{Tribe}_{own}$  well-formed types and heaps.

formulation [32] or using class tables [19, 42]. Finally,  $\mathcal{A}_{\mathcal{H}}$  finds the absolute type of an address in the heap.

Rules for type checking  $\text{Tribe}_{own}$  are given in Fig. 10. (T-SUBS), (T-LET), and (T-NULL) are standard. A variable (or **world**) type checks (T-VAR) if it is a valid type. A variable's minimal type is that variable itself; the type recorded in the environment can be used by subsumption. (T-FIELD), (T-ASSIGN), and (T-INVK) lookup the absolute type of the receiver and use that absolute type to look up the type of the field or method; otherwise, they are mostly standard. Of note in these rules is the substitution of the receiver for **this** (and similarly for the arguments in (T-INVK)), **this** (and arguments) may appear in  $\text{Tribe}_{own}$  types and must be substituted away accordingly. (T-NEW) requires a receiver which is unusual for object-oriented languages without nested classes: it allows the precise class to be instantiated to be determined statically. We require receivers and actual parameters to method calls to be variables, this simplifies substitution. We do not lose expressivity because we can use let expressions to create local variables as needed. If we create a local variable with a path, then we can use that

---


$$\frac{\mathcal{P} = \bar{Q} e}{\mathcal{P}(\mathbf{world}) = \bar{Q}} \quad \frac{\alpha \text{ class } C\langle\bar{x}\rangle \dots \{\bar{Q} \dots\} \in \mathcal{P}(A)}{\mathcal{P}(A.C\langle\bar{T}\rangle) = [\bar{T}/\bar{x}]\bar{Q}}$$

$$\frac{\vdash A \sqsubseteq_i^* A'.C\langle\bar{T}\rangle \quad \alpha \text{ class } C\langle\bar{x}\rangle \dots \{\bar{Q} \bar{U} \bar{f}; \bar{M}\} \in \mathcal{P}(A')}{fType(\bar{f}_k, A) = [\bar{T}/\bar{x}]\bar{U}_k}$$

$$\frac{\vdash A \sqsubseteq_i^* A'.C\langle\bar{T}\rangle \quad \alpha \text{ class } C\langle\bar{x}\rangle \dots \{\bar{Q} \bar{U}' \bar{f}; \bar{M}\} \in \mathcal{P}(A') \quad U m(\bar{U} \bar{x}) \{\mathbf{return } e;\} \in \bar{M}}{mType(m, A) = [\bar{T}/\bar{x}]\Pi \bar{x}:\bar{U}.U \quad mBody(m, A) = (\bar{x}; [\bar{T}/\bar{x}]e) \quad method(M_k, A)}$$

$$\frac{}{\mathbf{public}(\mathbf{world})}$$

$$\frac{\vdash A \sqsubseteq_i^* A' \quad \mathbf{public} \text{ class } C\langle\bar{x}\rangle \dots \in \mathcal{P}(A')}{\mathbf{public}(A.C\langle\bar{T}\rangle)}$$

$$\frac{}{\mathcal{A}_{\mathcal{H}}(\mathbf{world}) = \mathbf{world}}$$

$$\frac{\mathcal{H}(\iota) = \{r.N; \dots\}}{\mathcal{A}_{\mathcal{H}}(\iota) = \mathcal{A}_{\mathcal{H}}(r).N}$$

**Figure 9.** Auxiliary functions for  $\text{Tribe}_{own}$ .

path as a type so the type system can be aware of the relation between variable and path.

Rules for type checking programs, classes, and methods are given in Fig. 11. The absolute type of **this** is kept up to date in the environment by (T-CLASS) and (T-PROG). Elements of a class (methods, field types, classes, super-classes, etc.) are checked to be well-formed. We also check that imported modules are in fact modules, that is, they do not have an **out** field. We check that overriding methods have matching types with all methods they override. Method type checking is standard, other than that return and argument types may include the names of formal arguments and **this**.

**Operational semantics** We give a large step operational semantics for  $\text{Tribe}_{own}$  in Fig. 12; rules for handling error conditions are standard and are relegated to the accompanying technical report [15]. Despite the complexities of virtual class languages and ownership, all semantics rules are standard for object-oriented formalisms. In (R-NEW), we initialise fields to **null** and set the **out** field to the new object's surrounding object.

### 3.4 Properties

We prove preservation (subject reduction) for  $\text{Tribe}_{own}$ , this states that reduction preserves the type of expressions:

---

**Expression typing**  $\boxed{\Gamma \vdash e : T}$ 

$$\begin{array}{c}
\frac{\Gamma \vdash e : T'}{\Gamma \vdash e : T} \\
\text{(T-SUBS)}
\end{array}
\qquad
\frac{\Gamma \vdash e : T' \quad \Gamma, \mathbf{x} : T' \vdash e' : T}{\Gamma \vdash \mathbf{let } \mathbf{x} : T' = e \text{ in } e' : T} \\
\text{(T-LET)}$$

$$\frac{\Gamma \vdash \gamma \uparrow A \quad fType(\mathbf{f}, A) = T}{\Gamma \vdash \gamma.\mathbf{f} : [\gamma/\mathbf{this}]T} \\
\text{(T-FIELD)}$$

$$\frac{\Gamma \vdash \gamma \uparrow A \quad fType(\mathbf{f}, A) = T \quad \Gamma \vdash \gamma \uparrow A \quad \Gamma \vdash \gamma : [\gamma/\mathbf{this}, \gamma/\mathbf{x}]T}{\Gamma \vdash \gamma.\mathbf{m}(\bar{\gamma}) : [\gamma/\mathbf{this}, \gamma/\mathbf{x}]T} \\
\text{(T-INVK)}$$

$$\frac{\Gamma \vdash T \text{ OK}}{\Gamma \vdash \mathbf{null} : T} \\
\text{(T-NUL)}$$

$$\frac{\Gamma \vdash \gamma \uparrow A \quad \Gamma \vdash \gamma : [\gamma/\mathbf{this}, \gamma/\mathbf{x}]T}{\Gamma \vdash \mathbf{new } \gamma.N : \gamma.N} \\
\text{(T-NEW)}$$

$$\frac{\Gamma \vdash \gamma \uparrow A \quad \Gamma \vdash \gamma : [\gamma/\mathbf{this}, \gamma/\mathbf{x}]T \quad mType(\mathbf{m}, A) = \Pi \mathbf{x} : \bar{T}. T}{\Gamma \vdash \gamma.\mathbf{m}(\bar{\gamma}) : [\gamma/\mathbf{this}, \gamma/\mathbf{x}]T} \\
\text{(T-ASSIGN)}$$

$$\frac{\Gamma \vdash T \text{ OK}}{\Gamma \vdash s \text{ OK}} \\
\text{(T-VAR)}$$

**Figure 10.**  $\text{Tribe}_{own}$  expression typing rules.

**Theorem: preservation**  $\vdash \mathcal{H} \text{ OK}, \mathcal{H} \vdash e : T, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}', e' \neq \mathbf{err} \Rightarrow \vdash \mathcal{H}' \text{ OK}, e' = v, \mathcal{H}' \vdash v : T$ .  
Proof is by structural induction on the derivation of  $e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}'$ .

The proof of this theorem and supporting lemmas are given in the accompanying technical report [15]; the proof structure mostly follows [19].

#### 4. Tribal Topologies

In this section we will show how  $\text{Tribe}_{own}$  can be used as a descriptive [14, 16] ownership system ‘for free’ — that is, without any additional program annotations or formal rules. In the next section we will show how a range of different prescriptive ownership policies can be supported by the same underlying descriptive  $\text{Tribe}_{own}$  type system — separating ownership policies from the mechanisms that implement them [1].

The key insight is that the nesting of classes in objects, used in  $\text{Tribe}_{own}$  to support family polymorphism, is precisely the information needed to support descriptive ownership. Lexically, classes are nested in other classes; semantically, classes, and their instantiations, are nested under an

---

**Program typing**  $\boxed{\vdash \mathcal{P} \text{ OK}}$ 

$$\frac{\mathbf{this}:\mathbf{world} \vdash \bar{Q} \text{ OK} \quad \mathbf{this}:\mathbf{world} \vdash e : T \quad \sqsubseteq_i^+ \text{ is acyclic}}{\vdash \bar{Q} e \text{ OK}} \\
\text{(T-PROG)}$$

**Class typing**  $\boxed{\Gamma \vdash Q \text{ OK}}$ 

$$\frac{\Gamma(\mathbf{this}) = A' \quad \Gamma' = \Gamma[\mathbf{this} \mapsto A'.\mathbf{C}\langle\bar{\mathbf{x}}\rangle], \bar{\mathbf{x}} \quad \Gamma' \vdash A'.\bar{N}, \bar{A}, \bar{Q}, \bar{T}, \bar{M} \text{ OK} \quad \forall A \in \bar{A}. fType(\mathbf{out}, A) \text{ undefined} \quad \mathbf{f}_i = \mathbf{out} \Rightarrow T_i = \mathbf{this}.\mathbf{out} \quad \forall M, M' \in \{M \mid \text{method}(M, A'.\mathbf{C}\langle\bar{\mathbf{x}}\rangle)\}. \text{override}(M, M') \quad \alpha = \mathbf{private} \Rightarrow \forall A \text{ st } \vdash A'.\mathbf{C}\langle\bar{\mathbf{x}}\rangle \sqsubseteq_i^* A. \neg \text{public}(A)}{\Gamma \vdash \alpha \text{ class } \mathbf{C}\langle\bar{\mathbf{x}}\rangle \triangleleft \bar{N} \nabla \bar{A} \{ \bar{Q} \bar{T} \bar{\mathbf{f}}; \bar{M} \} \text{ OK}} \\
\text{(T-CLASS)}$$

**Method typing**  $\boxed{\Gamma \vdash M \text{ OK}}$ 

$$\frac{\Gamma, \bar{\mathbf{x}} : \bar{T} \vdash e : T \quad \Gamma, \bar{\mathbf{x}} : \bar{T} \vdash \bar{T}, T \text{ OK}}{\Gamma \vdash T \mathbf{m}(\bar{T} \bar{\mathbf{x}}) \{ \mathbf{return } e; \} \text{ OK}} \\
\text{(T-METHOD)}$$

$$\frac{M = T \mathbf{m}(\bar{T} \bar{\mathbf{x}}) \{ \mathbf{return } e; \} \quad M' = T \mathbf{m}(\bar{T} \bar{\mathbf{x}}) \{ \mathbf{return } e'; \}}{\text{override}(M, M')} \\
\text{(T-OVERRIDE)}$$

$$\frac{M = T \mathbf{m}(\bar{T} \bar{\mathbf{x}}) \{ \mathbf{return } e; \} \quad M' = T' \mathbf{m}'(\bar{T}' \bar{\mathbf{x}}') \{ \mathbf{return } e'; \} \quad \mathbf{m} \neq \mathbf{m}'}{\text{override}(M, M')} \\
\text{(T-OVERRIDE-NE)}$$

**Figure 11.**  $\text{Tribe}_{own}$  typing rules for classes and methods.

object: the owner of those instantiations. This nesting of objects is our ownership hierarchy.

The owner of an object (or an approximation to the owner) can be derived from its type. For object family types, the owner is the path part, for example, the owner of  $\mathbf{x}.\mathbf{C}$  is  $\mathbf{x}$ , the owner of  $\mathbf{this}.\mathbf{out}.\mathbf{C}$  is  $\mathbf{this}.\mathbf{out}$ , and the owner of  $\mathbf{world}.\mathbf{C}$  is  $\mathbf{world}$ . Variables with singleton (path) types can only hold the object indicated by the singleton type, therefore the owner of objects with singleton type is the owner of the path; e.g., the owner of objects with type  $\mathbf{x}.\mathbf{out}$  is  $\mathbf{x}.\mathbf{out}.\mathbf{out}$ .

If the type is a variable, then we can find a more precise owner by examining the type of the variable. For example, if  $\mathbf{x}$  has type  $\mathbf{y}.\mathbf{C}$ , then an object with type  $\mathbf{x}$  will have owner  $\mathbf{y}$  or  $\mathbf{x}.\mathbf{out}$ , and these two paths will always denote the same object. We know they denote the same object because they are subtypes ( $\mathbf{y}$  is a subtype of  $\mathbf{x}.\mathbf{out}$  by rules (S-

**Computation**  $\boxed{e; \mathcal{H} \rightsquigarrow v; \mathcal{H}}$

$$\begin{array}{c}
\frac{}{v; \mathcal{H} \rightsquigarrow v; \mathcal{H}} \quad \frac{\mathcal{H}(\iota) = \{T; \overline{\mathbf{f} \rightarrow v}\}}{\iota.\mathbf{f}_k; \mathcal{H} \rightsquigarrow v_k; \mathcal{H}} \\
\text{(R-VAL)} \quad \text{(R-FIELD)} \\
\frac{e; \mathcal{H} \rightsquigarrow v; \mathcal{H}' \quad [v/\mathbf{x}]e'; \mathcal{H}' \rightsquigarrow v'; \mathcal{H}''}{\mathbf{let } \mathbf{x}:T = e \text{ in } e'; \mathcal{H} \rightsquigarrow v'; \mathcal{H}''} \\
\text{(R-LET)} \\
\frac{\overline{\mathbf{f}} = \{\mathbf{f} \mid fType(\mathbf{f}, \mathcal{A}_{\mathcal{H}_n}(r).N) \text{ defined}\}}{\mathcal{H}' = \mathcal{H}_n, \iota \rightarrow \{r.N; \mathbf{out} \rightarrow r, \overline{\mathbf{f} \rightarrow \mathbf{null}}\}} \\
\mathbf{new } r.N; \mathcal{H} \rightsquigarrow \iota; \mathcal{H}' \\
\text{(R-NEW)} \\
\frac{e; \mathcal{H} \rightsquigarrow v; \mathcal{H}_1 \quad \mathcal{H}_1(\iota) = \{R; \overline{\mathbf{f} \rightarrow v}\}}{\mathcal{H}' = \mathcal{H}_1[\iota \mapsto \{R; \overline{\mathbf{f} \rightarrow v}[\mathbf{f}_k \mapsto v]\}]} \\
\iota.\mathbf{f}_k = e; \mathcal{H} \rightsquigarrow v; \mathcal{H}' \\
\text{(R-ASSIGN)} \\
\frac{e; \mathcal{H} \rightsquigarrow r; \mathcal{H}_n \quad mBody(\mathbf{m}, \mathcal{A}_{\mathcal{H}_n}(\iota)) = (\overline{\mathbf{x}}; e)}{[\iota/\mathbf{this}, r/\overline{\mathbf{x}}]e; \mathcal{H}_n \rightsquigarrow v; \mathcal{H}'} \\
\iota.\mathbf{m}(\overline{\mathbf{e}}); \mathcal{H} \rightsquigarrow v; \mathcal{H}' \\
\text{(R-INVOKE)}
\end{array}$$

**Figure 12.**  $\text{Tribe}_{own}$  reduction rules.

VAR) and (S-OUT-NEST)) and *path types* that are subtypes always denote the same object (which is obvious since their interpretation are singleton sets, and the interpretation of subtyping is subset).

For other types, we cannot say what the precise owner is. This is clear if we consider, for example, objects with type  $\mathbf{x}.\text{Course}.\text{RawMark}$  that denote any  $\text{RawMark}$  object in any  $\text{Course}$  object in  $\mathbf{x}$ . We say that the *owner type* of  $\mathbf{x}.\text{Course}.\text{RawMark}$  is  $\mathbf{x}.\text{Course}$ .

In general, an owner type in  $\text{Tribe}_{own}$  can be any non-variable type; if the owner type is not a path, then it does not denote a specific owner. Non-path owner types represent partial information about the actual owner, and are similar to existential [14, 49] or variant owners [35]. Subtyping between owner types can be used to identify types that refer to the same owner or set of owners, or refer to a more precise owner type. For example, if  $\mathbf{x}$  is a subtype of  $\mathbf{y}.\mathbf{C}$  then the owner type  $\mathbf{x}$  is a single owner and  $\mathbf{y}.\mathbf{C}$  describes a group of owners which includes  $\mathbf{x}$ .

An owner type is not defined for all types. Because module classes can be imported into any other class, objects with module type may have any owner. Therefore, types which indicate a module class do not have defined owner types; for example,  $\mathbf{world}.\text{Collections}$  cannot be assumed to have owner  $\mathbf{world}$ , because it may be imported into some other

**Owner relation**  $\boxed{\Gamma \vdash P \Downarrow P}$

$$\frac{\Gamma \vdash P \Uparrow A \quad fType(\mathbf{out}, A) \text{ defined}}{\Gamma \vdash P \Downarrow [P/\mathbf{x}]\mathbf{x}.\mathbf{out}}$$

(O-OWNER)

**Inside relation**  $\boxed{\Gamma \vdash P \prec: P}$

$$\begin{array}{c}
\frac{\Gamma \vdash P \prec: P' \quad \Gamma \vdash P' \Uparrow A \quad fType(\mathbf{out}, A) \text{ defined}}{\Gamma \vdash P \prec: P'} \quad \frac{\Gamma \vdash P \prec: P'' \quad \Gamma \vdash P'' \prec: P'}{\Gamma \vdash P \prec: P'} \\
\text{(I-SUB)} \quad \text{(I-TRANS)} \\
\frac{}{\Gamma \vdash p \prec: p.\mathbf{out}} \quad \frac{}{\Gamma \vdash P.N \prec: P} \\
\text{(I-OUT)} \quad \text{(I-CLASS)} \\
\frac{\mathcal{H}(\iota) = \{r.N; \dots\}}{\mathcal{H} \vdash \iota \prec: r} \\
\text{(I-RUNTIME)}
\end{array}$$

**Figure 13.**  $\text{Tribe}_{own}$  owner function and inside relation.

class with a different owner; if  $\mathbf{world}.\text{Collections}$  is imported into some non-module class  $\mathbf{C}$  with owner  $\mathbf{x}$  then  $\mathbf{x}.\text{Collections}$  will have owner  $\mathbf{x}$ , as usual. Note also that  $\mathbf{world}$  does not have an owner, because it is the top level of nesting.

We use the judgement  $\Gamma \vdash T \Downarrow P$  to find the owner ( $P$ ) from a type ( $T$ ). The owner judgement is defined in Fig. 13. Within a class,  $\mathbf{out}$  can be used to name the current object's owner in types and expressions. We find the owner by leveraging  $\text{Tribe}_{own}$  substitution: we attempt to append  $\mathbf{out}$  and  $\text{Tribe}_{own}$  substitution gives us the result described informally above. We must, of course, check that  $\mathbf{out}$  is defined in the given type.

At runtime, we can find an object's owner by observing its object record in the heap. The object record stores an object's runtime type which is an object family type, the owner can be found in the usual way. The owner can also be found by looking at the value in the object's  $\mathbf{out}$  field, these two approaches will give the same result in a well-formed heap.

By observing that each object in the heap is defined as having a single owner, and that the resulting owner relation between objects in the heap is acyclic (because the owner must be present in the heap at the time an object with that owner is created), we deduce that owners in the heap form a tree. Therefore, our ownership system describes an hierarchical topology for the heap. We prove below that the topology described statically by the type system is sound with respect to the runtime heap topology.

## 4.1 The Inside Relation

We define the inside relation [40] for owner types in  $\text{Tribe}_{own}$  in Fig. 13. The inside relation is reflexive (given by (I-SUB) and (S-REFLEX)) and transitive. Since subtyping indicates more precise descriptions of the same owner-type, subtyping between two owner types gives an inside relation<sup>4</sup>. Rules (I-OUT) and (I-CLASS) describe stepping outward or inward one step in the nested class hierarchy, which corresponds to one step outward or inward in the ownership hierarchy. (I-RUNTIME) is used to compute the inside relation for dynamic owner types and is based directly on the structure of the heap.

## 4.2 Imported Module Classes

Having owners defined by the lexical nesting of classes could limit reuse, for example, a programmer would need to write a new list class nested inside every class which needs to own a list. Reuse of classes by importing modules is thus crucial for creating a practical ownership system. Importing a module not only allows the code to be reused, but also allows the importing class to own the imported classes. To the best of our knowledge, this feature is not found in other virtual classes languages. For example,

```

module Collections {
  class Map<K, V> {...}
}

class University {
  ...
  class Course imports Collections{
    ...
    this.Map<this.out.Student,
      this.RawMark> marks_map = ...;
  }
}

```

Here, the field `marks_map` is an instance of `Map` owned by the current instance of `Course`.

## 4.3 Discussion

In most ownership systems, an object may be owned by any other object in the program and the owner is independent of the object's class. Classes are declared in a flat topology and there is no direct indication of the ownership hierarchy in the source code. In  $\text{Tribe}_{own}$ , an object's place in the ownership hierarchy is determined in part by the hierarchy of nested classes. Although the owner of each object may be different, the class of the owner is determined by the owned object's class. That is, while objects of the same class may be owned by different owners, the objects will be on the same tier of the ownership hierarchy and their owners will have a single class. Furthermore, objects of different classes are

<sup>4</sup>In fact, it indicates a 'reflexive' relationship with respect to the inside relation, if we offered a 'single step' (irreflexive, intransitive) version of the inside relation, then the rule (I-SUB) could not be used.

guaranteed to have different owners. Analogously to variance annotations [13], more of the ownership information in  $\text{Tribe}_{own}$  is defined where a class is declared, compared to other ownership systems where all ownership information is defined where classes are used.

In  $\text{Tribe}_{own}$  the nesting of classes specifies the topology of objects in the same way as individual classes specify the structure and behaviour of individual objects in class-based object-oriented languages.

## 4.4 Tribal Effects

To show that our descriptive ownership system is useful, we have used it to define an effect system [45] for  $\text{Tribe}_{own}$ . The nested-ness of  $\text{Tribe}_{own}$  is used to define computational effects without extra ownership or regions notations. This should make effects systems accessible and attractive to programmers. We offer the formalised effect system without proofs in our technical report [15].

Our effects can describe a single object (by using a singleton type and an 'exact' annotation), a single object and its transitive representation (by using a singleton type), a context (exact, non-singleton type), or a context and all contexts under it (non-singleton types). Effects can be combined. The sub-effect relation follows from the subtype and inside relations. Our effect assignment rules are mostly standard and we expect the usual rules for disjointness of effects [18] to apply. Therefore, Tribal effects could be used to support parallelisation of  $\text{Tribe}_{own}$  code [18, 5].

## 4.5 Properties

We prove that ownership in  $\text{Tribe}_{own}$  is sound; that is, the owner given by an expression's static type corresponds to the owner, found in the heap, after evaluating that expression. This result is necessary for supporting an effect system or other application of descriptive ownership.

**Theorem: Reduction preserves owners**  $\vdash \mathcal{H} \text{ OK}, \mathcal{H} \vdash e : T, e; \mathcal{H} \rightsquigarrow \iota; \mathcal{H}', \mathcal{H} \vdash T \Downarrow P \Rightarrow \mathcal{H}'(\iota) = \{r.C<...>; \dots\}, \mathcal{H}' \vdash r <: P$

This theorem follows straightforwardly from our preservation theorem (see Sect. 3.4) and the following lemma.

**Lemma: Subtyping preserves owners**  $\Gamma \vdash P <: P', \Gamma \vdash P' \Downarrow P'_0 \Rightarrow \Gamma \vdash P \Downarrow P_0, \Gamma \vdash P_0 <: P'_0$ . Proof is by structural induction on the derivation of the subtyping judgement.

In  $\text{Tribe}_{own}$  different paths can point to the same object, thus a single owner can be denoted by different types. We formalise this fact using subtyping:  $p$  and  $p'$  denote the same object if and only if  $p$  is a subtype of  $p'$  or  $p'$  is a subtype of  $p$ . This statement only makes sense if subtyping monotonically increases precision of types, we prove this as a lemma to support our ownership soundness argument:

**Lemma: Subtypes are more precise**  $\Gamma \vdash T <: p \Rightarrow T = p'$  and  $\Gamma \vdash T <: \gamma.N \Rightarrow T \in \{p, p.N'\}$ . Proof is by structural induction on the derivation of the subtyping judgement.

## 5. Encapsulation

In this section, we show how encapsulation policies can be enforced in  $\text{Tribe}_{own}$  by building on top of the descriptive ownership outlined in the previous section. We construct three different variants of owners-as-modifiers (in Fig. 14) and a new, more flexible variant of owners-as-dominators with two mechanisms for enforcement (in Fig. 15); we show how standard owners-as-dominators is a simple restriction of our variant. Encapsulation policies in  $\text{Tribe}_{own}$  are defined independently of the mechanism of descriptive ownership, and are, essentially, pluggable.

### Owners-as-Modifiers (syntactic)

$$\frac{\Gamma \vdash \gamma : \mathbf{this}.N^* \quad \dots}{\Gamma \vdash \gamma.f = e : \dots \quad \Gamma \vdash \gamma.m(\bar{e}) : \dots} \text{(T-ASSIGN T-INVK)}$$

### Owners-as-Modifiers (expressive)

$$\frac{\Gamma \vdash \gamma \Downarrow P \quad \Gamma \vdash P <: \mathbf{this} \vee P = \mathbf{this.out} \quad \dots}{\Gamma \vdash \gamma.f = e : \dots \quad \Gamma \vdash \gamma.m(\bar{e}) : \dots} \text{(T-ASSIGN T-INVK)}$$

### Owners-as-Modifiers (Universes)

$$\frac{\Gamma \vdash \gamma \Downarrow P \quad P \in \{\mathbf{this}, \mathbf{this.out}\} \quad \dots}{\Gamma \vdash \gamma.f = e : \dots \quad \Gamma \vdash \gamma.m(\bar{e}) : \dots} \text{(T-ASSIGN T-INVK)}$$

**Figure 14.** Enforcing owners-as-modifiers in  $\text{Tribe}_{own}$ .

### 5.1 Owners-as-Modifiers

Owners-as-modifiers is an encapsulation policy first implemented in the Universes type system [39, 27, 23]. It enforces that any modification of an object must be initiated by its owner.

Owners-as-modifiers in the Tribal setting is implemented by adding restrictions (extra premises) to the rules for field assignment and method invocation. For simplicity, we ignore

### Owners-as-Local-Dominators

$$\frac{\Gamma \vdash \mathbf{this} \oplus \bar{T} \quad \dots}{\Gamma \vdash \alpha \text{ class } C \langle \bar{X} \rangle < \bar{A} \nabla \bar{C}' \{ \bar{Q} \bar{T} \bar{f}; \bar{M} \} \text{ OK}} \text{(T-CLASS)}$$

$$\frac{\Gamma \vdash P.C \langle \bar{T} \rangle \oplus \bar{T} \quad \dots}{\Gamma \vdash P.C \langle \bar{T} \rangle \uparrow A.C \langle \bar{T} \rangle} \text{(A-CLASS)}$$

### Owners-as-Local-Dominators Heap

$$\forall \iota \rightarrow \{R; \bar{f} \rightarrow v\} \in \mathcal{H}$$

$$\frac{\forall f \rightarrow v \in \bar{v}. f \neq \mathbf{out} \wedge v \neq \mathbf{null} \Rightarrow \Gamma \vdash \iota <: v.out \vee (\mathcal{H} \vdash \mathcal{H}(v) \uparrow A \wedge \mathit{public}(A)) \quad \dots}{\vdash \mathcal{H} \text{ OK}} \text{(F-HEAP)}$$

**Figure 15.** Enforcing owners-as-dominators in  $\text{Tribe}_{own}$ .

pure methods. They could easily be supported by allowing all calls to pure methods; the formalisation is standard (e.g., [23]).

Over the next few paragraphs we describe three variations of the owners-as-modifiers discipline. Each variation is implemented in a different way and enforces a slightly different invariant; all three variations are defined formally in Fig. 14.

**Syntactic variant** Our “syntactic variant” of owners-as-modifiers has a simple implementation, formalised by examining the syntax of the static type of the receiver. We only allow field assignment and method invocations on objects nested inside the current **this**.

The syntactic variant allows an aggregate to manipulate its (transitive) representation (instances of classes nested, possibly deeply, inside the aggregate), but not operate on peer objects (instances of classes directly nested inside the aggregate’s owner).

**Expressive variant** The “expressive variant” is, to the best of our efforts, the most flexible policy that sticks to the spirit of owners-as-modifiers. It allows access to transitively owned objects and peer objects. It is formalised using the owner and inside relations, rather than a syntactic test, and is therefore more expensive to implement. It handles paths which alias a transitively owned object. Finally, it subsumes the simple variant.

**Universes variant** The “Universes variant” emulates Universe Types. An object can modify its own representation and peer objects, but nothing else. The formalisation uses the

---

**Naming**  $\boxed{\Gamma \vdash P \oplus T}$

**Local o-as-d, semantic style**

$$\frac{\Gamma \vdash P' \Downarrow P'' \quad \Gamma \vdash P \prec: P''}{\Gamma \vdash P \oplus P'} \quad \frac{}{\Gamma \vdash P \oplus X}$$

(N-O-AS-D) (N-TYPE-VAR)

$$\frac{\Gamma \vdash T \Uparrow A \quad \text{public}(A.N) \quad \Gamma \vdash P \oplus T}{\Gamma \vdash P \oplus T.N}$$

(N-PUBLIC)

**Local o-as-d, syntactic style**

$$\frac{\Gamma \vdash T \Uparrow A \quad \text{public}(A.N) \quad \Gamma \vdash P \oplus T}{\Gamma \vdash P \oplus T.N} \quad \frac{}{\Gamma \vdash P \oplus X}$$

(N-PUBLIC) (N-TYPE-VAR)

$$\frac{\Gamma \vdash p.N \oplus p.N \quad \Gamma \vdash p \oplus p}{\Gamma \vdash p \oplus p} \quad \frac{}{\Gamma \vdash p \oplus p.N}$$

(N-REFLEX) (N-REP)

$$\frac{\Gamma \vdash P \oplus p \quad \Gamma \vdash P \oplus p.\text{out} \quad \Gamma \vdash P \oplus p.\text{out}.N}{\Gamma \vdash P \oplus p} \quad \frac{}{\Gamma \vdash p.N \oplus p}$$

(N-OUT-1) (N-OUT-2)

$$\frac{\Gamma \vdash P' \oplus T \quad \Gamma \vdash P \prec: P'}{\Gamma \vdash P \oplus T}$$

(N-SUB)

---

**Figure 16.**  $\text{Tribe}_{own}$  naming restrictions.

owner relation, but not the inside relation (and thus, not subtyping), it is therefore ‘more syntactic’ than the expressive variant, but not purely so. Similarly to the syntactic variant, the Universes variant only handles types which start at **this**, not equivalent types.

## 5.2 Owners-as-Local-Dominators

In a system that supports owners-as-dominators, an object’s owner is found on all access paths from the root of the system to the object itself [22]. In this section we propose a new, more flexible variation of owners-as-dominators which has two formalisations. We call our policy owners-as-local-dominators; it subsumes owners-as-dominators.

The rules in figures 15 and 16 define owners-as-local-dominators encapsulation in  $\text{Tribe}_{own}$ . By (T-CLASS), a class may only use types that it is allowed to name. Naming, denoted  $\oplus$ , is defined in two ways: a *semantic variant* with few, declarative rules and a *syntactic variant* with more rules, but using only the syntax of types, where possible.

We call our policy owners-as-local-dominators because it enforces owners-as-dominators over local sub-heaps, rather than the whole heap. This is supported in the language by declaring classes **public** or **private**. Public class names can be named anywhere in a program, private class names can only be named within the surrounding class’s declaration (at any level of nesting). These private naming rules mimic standard ownership types, where naming is dictated by the visibility of **this**. As a consequence, each instance of a private class forms the root of an encapsulated sub-heap. Objects in the sub-heap may be referenced only according to the owners-as-dominators policy.

In both our naming formalisations, the rules for public classes and type variables are the same. Both sets of rules handle aliasing paths by subtyping, in the semantic version via (I-SUB), and explicitly in the syntactic version.

A public class nested inside an encapsulated sub-heap may be referenced freely *within that heap*, but not outside. It is easy to see why this is the case: WLOG, assume  $p.C$  is a type where  $p$  is a path to an object with private class and  $C$  is a public class name. By the rules in Fig. 16, to name  $p.C$ , an object must be able to name  $p$ . As a result, public classes provide flexibility without compromising safety for objects inside an aggregate. Importantly, it is not possible to define a public class inside an encapsulated sub-heap and use it as a proxy to export references to objects outside the sub-heap<sup>5</sup>.

For concreteness, consider the following code:

```
private class A {
  private class B {
    private class C {}
    public class D {}
  }
  private class E {
    this.out f1; //OK - owner
    this.out.B f2; //OK - peer
    this.out.B.C f3; //ERROR - private class
    this.out.B.D f4; //OK - can name B
    world.F f5; //OK - public class
  }
}
public class F {
  world.A f6; //OK - peer
  world.A.B.D //ERROR can't name B
}
```

Public classes may inherit from private classes, but not vice versa. Therefore, instances of private classes cannot be named by subsumption. It is safe to subsume a public class to a private class because a public class can be named everywhere that a private class can.

Because we wish to use `Root.Object` as a top type, we must be able to name it everywhere, and so it must be a public class. Since private classes can not inherit from public classes, we require a `Root.PrivateObject` to be the root of

<sup>5</sup> This distinguishes deep ownership from shallow ownership.

the inheritance hierarchy for private classes<sup>6</sup>. Unfortunately, this means `Root.Object` is not a top type after all, but only one for public classes. We believe this is a feature rather than a flaw, as subsuming private classes to a global top type effectively breaks encapsulation.

If all classes are private (or we remove the rule (N-PUBLIC)), our system upholds the owners-as-dominator property. If all classes are public, then we enforce no encapsulation policy and have descriptive ownership. By using a mix of public and private classes, we allow the programmer to define the level of encapsulation required.

For increased flexibility we only enforce our encapsulation policy *on the heap*. Similar to Clarke and Drossopoulou’s `Joe1` [18], we do not enforce naming restrictions on the types of local variables. (See also a discussion of borrowing and owner-polymorphic methods in [20].)

Owners-as-local dominators and a default **public** annotation for classes makes it easy to allow arbitrary aliasing in a system; except for specific sub-heaps which can be easily created by a single **private** annotation on a class. This is similar to the flat ownership hierarchies in Joelle [21] and Loci [50], where fields are implicitly encapsulated in the current owning object, and **active** and `@Thread` annotations are used as “cut-off points” to define the root of an encapsulated sub-heap. The big difference is that in `Tribeown`, the ownership hierarchy is not flat, even when aliasing is permitted, which facilitates gradually increasing encapsulation.

### 5.2.1 Properties

We define  $oasld(\mathcal{H})$  if  $\forall \iota, \iota' \in \mathcal{H}$  where  $\mathcal{H} \vdash \iota \leftrightarrow \iota'$ .  $\Gamma \vdash \iota \prec: \iota'.out \vee (\mathcal{H} \vdash \mathcal{H}(\iota') \uparrow A \wedge public(A))$ , where  $\leftrightarrow$  means has a reference to.

**Theorem: owners-as-local-dominators**  $\vdash \mathcal{H} \text{ OK}, \mathcal{H} \vdash e : T, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}', oasld(\mathcal{H}) \Rightarrow oasld(\mathcal{H}')$

We prove this theorem for both syntactic and semantic variants of our naming relation. Both are proved as part of our preservation theorem by proving that the extra invariants in (F-HEAP) are preserved. This requires extending our substitution lemmas to the inside, owner, and naming relations, and, in the syntactic case, proving that syntactic naming gives semantic naming.

### 5.2.2 Discussion

Superficially, owners-as-local-dominators appears to be orthogonal to the family-polymorphic aspects of Tribal ownership. The programmer can specify encapsulated sub-heaps by annotating classes, however, only because of the unique correspondence between the nested class hierarchy and the ownership hierarchy. In a context-parametric ownership system, it would only make sense to annotate individual context

<sup>6</sup>`PrivateObject` can be inherited from because its surrounding class is public. `Root.PrivateObject` cannot be used as a supertype of the local `PrivateObjects` because it cannot be named.

parameters. This is not a solution however, because we could not control the nesting of public and private contexts; therefore, this would not create encapsulated sub-heaps.

## 6. Related Work

`Tribeown` is closest in spirit to work by Dietl and Müller [26] expressing ownership type systems on top of dependent classes (a similar language feature to virtual classes, but that allows classes to be nested inside more than one enclosing object). Their system is inspired by the original Tribe work and has similar properties to `Tribeown`—topological constraints and encapsulation restrictions are handled separately. Their system is an encoding and has not been formalised or proven correct. Furthermore, it requires more annotations (notably, special owner annotations) than our system. Thus, we believe that `Tribeown` is a further simplification of Dietl and Müller’s system. `Tribeown` also comes with proofs of type and ownership soundness.

The simplest way to support ownership in a Tribe-like language is to mimic the context parameters of context-parametric ownership systems with fields. For example, a non-generic map could be written as:

```
class Map {
  Object keyOwner;
  Object valueOwner;
  this.Node[] nodes;

  Map(Object keyOwner, Object valueOwner) {...}

  class Node {
    this.out.keyOwner.Object key;
    this.out.valueOwner.Object value;
  }
}

Map m = new Map(k, v);
k.Object ko = new k.Object();
m.put(ko, new v.Object()); //type error
v.Object vo = m.get(ko); //type error
```

Although, this code looks sensible, it cannot be type checked because the type system cannot determine that `m`’s `keyOwner` is the same object as `k`, and similarly for `valueOwner`. This makes the map essentially useless if we want to preserve ownership information. We address this problem in `Tribeown` by using generics. In a fully dependent class system such as Dietl and Müller’s, `Map` can be dependent upon `keyOwner` and `valueOwner`, as well as the map’s owner, allowing programmers to write this kind of code.

`Tribeown` uses similar restrictions to Confined Types [31]—a simple set of rules to guarantee that instances of package scoped classes defined in a package are not referenced outside the package. In confined type, there is no nesting or notion of topology. In this respect, the encapsulation pro-

vided by confined types is similar to systems that offer flat ownership hierarchies, like Joelle [21] and Loci [50].

**Ownership and Alias Management Systems** There are a plethora of ownership systems to date, notably Clarke’s original deep ownership system [22], Universes [39], shallow ownership [3], and Ownership Domains [1]. Ownership Domains was the first system to advocate separating policy from topology. Ownership Domains is more flexible than Tribe, but requires more annotations. Since the Ownership Domains programmer can customise encapsulation, the type system will only enforce ad hoc encapsulation, not an encapsulation policy. Therefore, compilers and programmers cannot rely on the invariants of a policy to aid reasoning. In similar spirit, Boyland et al. [11] define a set of capabilities on pointers, although without a static type system.

Tribe<sub>own</sub>’s **out** field is similar to Pedigree Types’ chains of parents for transitive owners, although Pedigree Types are not based on virtual or nested classes [34].

Inner classes in Java are a primitive kind of nested class, lacking virtual inheritance. Boyapati et al. [8] have proposed integrating inner classes with a typical context-parametric ownership types system. In their system, the ownership hierarchy is defined orthogonally to the nested class hierarchy: inner classes have owners independent of their surrounding classes. Their system supports a modified owner-as-dominators encapsulation policy where, as well as the usual rules, inner classes have privileged access to the representations of their enclosing objects. Our owners-as-dominators policy is similar to the inner classes part of Boyapati et al.’s policy. Our policy is more flexible because nested classes have access to their peer and owned classes, as well as their owners, and more intuitive because of the coincidence of the ownership and nested class hierarchies. Our scheme of public classes introduces further flexibility so we avoid the need for having a separate ownership hierarchy.

**Virtual Class Systems** Since Ernst’s original proposal [29], built on gBeta [28], a range of different proposals for virtual class systems have appeared, both with *class families* (Concord [33], .FJ [44], and Jx [41]) and *object families* (Caesar/J [38], vc [30], Scala [36], gBeta [28], and Tribe). For ownership types, object families are needed since encapsulation is per-object. Building an ownership system on top of Scala is an interesting direction for future work.

Recently, Bach Nielsen and Ernst have investigated VM support for virtual classes [4]. We believe that having efficient VM support for virtual classes could enable space-efficient downcasting in ownership types, which is a long-standing problem in the community (see further [6, 49]).

Bracha’s Newspeak [12] is a recent, dynamically typed language with virtual classes. Like the aforementioned languages, Newspeak does not have ownership support.

## 7. Conclusion and Further Work

We have shown how Tribe<sub>own</sub>, a language with virtual classes, can support ownership types, and a range of different encapsulation policies, in a flexible and straightforward way. We have shown that using a class’s enclosing object as the owner of instances of that class gives a sound ownership system. This has been suggested before (first, to our knowledge, by Clarke [17], later by Dietl and Müller [26]) but not developed, formalised, or proven. A language based on Tribe<sub>own</sub> will have low syntactic overhead, and should make it easy for programmers to comprehend the ownership hierarchy. We have further described a novel, flexible encapsulation policy, owners-as-local-dominators; an extension to the standard owners-as-dominators policy, which leverages the nested-class-based ownership of Tribe<sub>own</sub> and allows the programmer to customise the level of encapsulation enforced.

**Future work** We have shown how both an owners-as-modifiers policy and an owners-as-dominator policy can be layered on top of Tribe<sub>own</sub>, and how the latter can be localised. We did not find an obvious way to localise owners-as-modifiers. We hope to find a way to do this, then combine the local variants of both properties in the same program.

We believe that class invariants can be extended to families of classes to describe invariants which depend on the relationships between classes as well as the classes themselves; for example, to verify subject-observer [43].

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## A. Elided Figure

<b>Error Handling</b>	$e; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}$
$\frac{}{\text{null}.f; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-FIELD-NULL)</p>	
$\frac{}{\text{null}.f = e; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-ASSIGN-NULL)</p>	
$\frac{}{\lrcorner \text{new null}.N; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-NEW-NULL)</p>	
$\frac{}{\text{null}.m(\bar{e}); \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-VOKE-NULL-RECV)</p>	
$\frac{e; \mathcal{H} \rightsquigarrow v; \mathcal{H}_n \quad \text{null} \in \bar{v}}{\iota.m(\bar{e}); \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-VOKE-NULL-ARG)</p>	
$\frac{e; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}'}{\text{let } x:T = e \text{ in } e'; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-LET-ERR)</p>	
$\frac{e; \mathcal{H} \rightsquigarrow v; \mathcal{H}' \quad [v/x]e; \mathcal{H}' \rightsquigarrow \mathbf{err}; \mathcal{H}''}{\text{let } x:T = e \text{ in } e'; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-LET-ERR)</p>	
$\frac{e; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}'}{\iota.f = e; \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-ASSIGN-ERR)</p>	
$\frac{e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}' \quad \mathbf{err} \in e'}{\iota.m(\bar{e}); \mathcal{H} \rightsquigarrow \mathbf{err}; \mathcal{H}}$ <p style="text-align: center;">(R-VOKE-ERR)</p>	

**Figure 17.**  $\text{Tribe}_{own}$  reduction rules.

Rules for error handling in the operational semantics are given in Fig. 17.

## B. Proofs

### B.1 Supporting Lemmas

#### B.1.1 Weakening

1. **Absolute Types:**  $\Gamma_1, \Gamma_2 \vdash T \uparrow A, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset \Rightarrow \Gamma_1, \bar{x}:T, \bar{x}, \Gamma_2 \vdash T \uparrow A$ . Proof by s. ind., case analysis on last step:

**A-Var** second premise, ind hyp

**A-World** trivial

**A-Out** ind hyp

**A-Class** ind hyp

2. **Well-formedness:**  $\Gamma_1, \Gamma_2 \vdash T \text{ OK}, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset \Rightarrow \Gamma_1, \bar{x}:T, \bar{x}, \Gamma_2 \vdash T \text{ OK}$ . Proved as part of the abs type case:

**F-Path-Type** ind hyp

**F-Type-Var** by third premise

3. **Subtyping:**  $\Gamma_1, \Gamma_2 \vdash T_1 <: T_2, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset \Rightarrow \Gamma_1, \bar{x}:T, \bar{x}, \Gamma_2 \vdash T_1 <: T_2$ . Proof by s. ind., case analysis on last step:

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** absolute types case

**S-Nest-Class** ind hyp, well-formedness case

**S-Var** second premise

**S-Out-Nest** ind hyp, absolute types case

**S-Out-2** absolute types case

**S-Eq** second premise

**S-Import** abs types case

4. **Type checking:**  $\Gamma_1, \Gamma_2 \vdash e : T, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset, \bar{x} \cap \text{dom}(\Gamma_1, \Gamma_2) = \emptyset \Rightarrow \Gamma_1, \bar{x}:T, \bar{x}, \Gamma_2 \vdash e : T$ . Proof by s. ind., case analysis on last step:

**T-Subs** ind hyp, subtyping case, well-formedness case

**T-Field** absolute types case

**T-Assign** absolute types case, ind hyp

**T-New** absolute types case, ind hyp

**T-Null** well-formedness case

**T-Var** well-formedness case

**T-Invk** absolute types case, ind hyp

**T-Let** ind hyp, well-formedness case

#### B.1.2 Inversion

1. **Field access:**  $\Gamma \vdash \gamma.f : T \Rightarrow \Gamma \vdash \gamma \uparrow A, fType(f, A) = T', \Gamma \vdash [\gamma/\mathbf{this}]T' <: T$
2. **Field assignment:**  $\Gamma \vdash \gamma.f = e : T \Rightarrow \Gamma \vdash \gamma \uparrow A, fType(f, A) = T', \Gamma \vdash e : [\gamma/\mathbf{this}]T', \Gamma \vdash [\gamma/\mathbf{this}]T' <: T$
3. **Object creation:**  $\Gamma \vdash \text{new } \gamma.N(\bar{e}) : T \Rightarrow \Gamma \vdash p \uparrow A, \bar{f}\bar{f} = \{\bar{f}\bar{f} | fType(\bar{f}\bar{f}, A.N) \text{ defined}\}, fType(\bar{f}\bar{f}, A.N) = T, \Gamma \vdash e : [\gamma/\mathbf{this}]T, \Gamma \vdash \gamma.N <: T$
4. **Method invocation:**  $\Gamma \vdash \gamma.m(\bar{\gamma}) : T \Rightarrow \Gamma \vdash \gamma \uparrow A, \Gamma \vdash \bar{\gamma} : [\gamma/\mathbf{this}, \bar{\gamma}/\bar{x}]T, mType(m, A) = \Pi \bar{x}:T.T'', \Gamma \vdash [\gamma/\mathbf{this}, \bar{\gamma}/\bar{x}]T' <: T$
5. **Variables:**  $\Gamma \vdash s : T \Rightarrow \Gamma \vdash s \text{ OK}, \Gamma \vdash s <: T$

6. Let expressions:  $\Gamma \vdash \text{let } x:U = e \text{ in } e' : T \Rightarrow \Gamma \vdash e : U, \Gamma, x:U \vdash e' : T', \Gamma \vdash T' \text{ OK}, \Gamma \vdash T' <: T$

All cases proved by s. ind. on typing derivation, applying ind hyp in the T-Subs case and by S-Reflex in the other case

### B.1.3 Inheritance preserves importability

$\vdash A \sqsubseteq_i^* A', fType(\mathbf{out}, A) \text{ undefined} \Rightarrow fType(\mathbf{out}, A') \text{ undefined}$ .  
Proof follows from def of  $fType$ .

### B.1.4 Properties of inheritance

1.  $\vdash A.N \sqsubseteq_i^* A'.N' \Rightarrow fType(\mathbf{out}, A'.N') \text{ undefined or } \vdash A \sqsubseteq_i^* A', \vdash A.N \sqsubseteq_s^* A.N'$  by s. ind., transitivity is the interesting case: by ind hyp, lemma B.1.3 SC-Nest, transitivity; SC-Import by T-Class; all other cases are by reflexivity, ind hyp, premise, or are trivial.

### B.1.5 Absolute types are deterministic

$\Gamma \vdash T \uparrow A, \Gamma \vdash T \uparrow A' \Rightarrow A = A'$

Proof by double s. induction:

**A-Var** ind hyp,  $T \neq \mathbf{world}$  since  $\Gamma(T) = T'$

**A-World** trivial

**A-Out** ind hyp

**A-Class** ind hyp

### B.1.6 Type Substitution

Let  $\Gamma = \Gamma_1, \bar{x}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\bar{T}/\bar{x}]\Gamma_2$  where  $fv(\Gamma_1) \cap \bar{x} = \emptyset$  and  $\Gamma_1 \vdash \bar{T} \text{ OK}$ .

1. class lookup:  $\mathcal{P}(A) = Q \Rightarrow \mathcal{P}([\bar{T}/\bar{x}]A) = [\bar{T}/\bar{x}]Q$ , proof by s. ind., case analysis etc.: **world** case is easy, ind case is by ind hyp, distinctness of formal variables, re-arrangement of substitutions.

2. inheritance:  $\vdash A \sqsubseteq_i^* A' \Rightarrow \vdash [\bar{T}/\bar{x}]A \sqsubseteq_i^* [\bar{T}/\bar{x}]A'$ , proof by s. ind., case analysis etc.:

**Reflexivity** trivial

**Transitivity** ind hyp

**I-SC** ind hyp

**I-FB** ind hyp

**SC-SubClass** by class lookup case, rearrangement of substitutions

**SC-Nest** ind hyp

**SC-Import** by class lookup case, rearrangement of substitutions

**FB-Nest** by class lookup case, rearrangement of substitutions

3. absolute types:  $\Gamma \vdash T \uparrow A \Rightarrow \Gamma' \vdash [\bar{T}/\bar{x}]T \uparrow [\bar{T}/\bar{x}]A$ , proof by s. ind., case analysis etc.:

**A-Var** easy, with a case analysis on  $x \in \Gamma_1$  or  $x \in \Gamma_2$ .

**A-World** trivial

**A-Out**  $[\bar{T}/\bar{x}]p = p$ , done by ind hyp, def  $fType$

**A-Class** ind hyp, inheritance case, class lookup case

4. well-formed types:  $\Gamma \vdash T \text{ OK} \Rightarrow \Gamma' \vdash [\bar{T}/\bar{x}]T \uparrow$ , proved with the above case, either by ind hyp (F-Path-Type), or by well-formedness of  $\bar{T}$  (F-Type-Var).

5. subtyping:  $\Gamma \vdash T <: T' \Rightarrow \Gamma' \vdash [\bar{T}/\bar{x}]T <: [\bar{T}/\bar{x}]T'$ , proof by s. ind., case analysis etc.:

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** abs type and inheritance cases

**S-Nest-Class** ind hyp

**S-Var** easy, with a case analysis on  $x \in \Gamma_1$  or  $x \in \Gamma_2$ .

**S-Out-Nest**  $[\bar{T}/\bar{x}][\mathbb{Q}/x]x.\mathbf{out} = [[\bar{T}/\bar{x}]\mathbb{Q}/x]x.\mathbf{out}$ , ind hyp, abs type case, def  $fType$

**S-Out-2** abs type case, def  $fType$

**S-Import** wf types lemma, previous lemma, wf prog, type substitution lemma

6.  $fType: fType(\mathbf{f}, A) = T \Rightarrow fType(\mathbf{f}, [\bar{T}/\bar{x}]A) = [\bar{T}/\bar{x}]T$  proof by inheritance and class lookup cases

7.  $mType: mType(\mathbf{m}, A) = \Pi \bar{x}:\bar{T}.T \Rightarrow mType(\mathbf{m}, [\bar{T}/\bar{x}]A) = [\bar{T}/\bar{x}]\Pi \bar{x}:\bar{T}.T$  proof by inheritance and class lookup cases

8. type checking:  $\Gamma \vdash e : T \Rightarrow \Gamma' \vdash [\bar{T}/\bar{x}]e : [\bar{T}/\bar{x}]T$ , proof by s. ind., case analysis etc.:

**T-Subs** ind hyp, subtyping case, well-formedness case

**T-Field** abs type case,  $fType$ case

**T-Assign** abs type case,  $fType$ case, ind hyp, re-arranging substitutions

**T-New** abs type case, def  $fType$ ,  $fType$ case, ind hyp, re-arranging substitutions

**T-Null** well-formedness case

**T-Var** trivial

**T-Invk** abs type case, ind hyp,  $mType$ case

**T-Let** ind hyp, well-formedness case

### B.1.7 Properties of inheritance, methods and fields

1. Inheritance preserves  $mType$ :  $mType(\mathbf{m}, A) = \Pi \bar{x}:\bar{T}.T, \vdash A' \sqsubseteq_i^* A, mType(\mathbf{m}, A') = \Pi \bar{x}':\bar{T}'.T' \Rightarrow \bar{x} = \bar{x}', \bar{T} = \bar{T}', T = T'$  Proof: by inversion of  $mType$  we find  $M$  and  $M'$  which give  $mType$  for  $A$  and  $A'$ , both are in  $methods(A')$  by definitions of  $methods$ . We are then done by T-Prog, T-Class, (and in the case of  $M$ , transitivity) and T-Override.

2. Inheritance preserves  $fType$ :  $fType(\mathbf{f}, A) = T, \vdash A' \sqsubseteq_i^* A, fType(\mathbf{f}, A') = T' \Rightarrow T = T'$  Proof as above

### B.1.8 Reflexivity of absolute types

$\Gamma \vdash A \text{ OK} \Rightarrow \Gamma \vdash A \uparrow A$  proof by easy s. induction on  $\Gamma \vdash A \uparrow T$

### B.1.9 Defined parts of a program are well-formed

$\mathcal{P}(A) \Rightarrow A \vdash \text{OK}$  Proof by s. induction on derivation of  $\mathcal{P}(A)$ , base case by A-World, ind case by ind hyp, reflexivity of abs types, A-Class.

### B.1.10 Inheritance and absolute types

**this**:  $A_0 \vdash T \uparrow A$ , **this**:  $A'_0 \vdash T \uparrow A'$ ,  $\vdash A_0 \sqsubseteq_i^* A'_0 \Rightarrow \vdash A \sqsubseteq_i^* A'$

Proof by double induction on **this**:  $A_0 \vdash T \uparrow A$ , **this**:  $A'_0 \vdash T \uparrow A'$  case analysis on the last step, noting that the rule applied in both cases is the same because the absolute types judgement is syntax directed by  $T$ :

**A-Var** lemma B.1.8

**A-World** trivial

**A-Out** ind hyp, lemma B.1.4

**A-Class** ind hyp, transitivity, reflexivity

### B.1.11 Inheritance gives subtyping

$\forall \Gamma : \vdash A \sqsubseteq_i^* A' \Rightarrow \Gamma \vdash A <: A'$

Proof by s. ind. on  $\vdash A \sqsubseteq_i^* A'$ , case analysis:

**Reflexivity** by S-Reflex

**Transitivity** by ind hyp, S-Trans

**I-SC** ind hyp

**I-FB** ind hyp

**SC-SubClass** lemma B.1.9, reflexivity of abs types, S-SubClass

**SC-Nest** lemma B.1.9, reflexivity of abs types, S-SubClass

**SC-Import** lemma B.1.9, A-Class, reflexivity of abs types, reflexivity of inheritance, S-Import

**FB-Nest** lemma B.1.9, reflexivity of abs types, A-Class, F-Type, S-Nest-Class

### B.1.12 Inheritance preserves absolute types

$\vdash A \sqsubseteq_i^* A', \Gamma \vdash A \uparrow A, \vdash \Gamma \text{ OK} \Rightarrow \Gamma \vdash A' \uparrow A'$

**Reflexivity** trivial

**Transitivity** ind hyp

**I-SC** ind hyp

**I-FB** ind hyp

**SC-SubClass** wf program, T-Class, type substitution lemma

**SC-Nest** ind hyp, def A-Class

**SC-Import** wf program, T-Class, type substitution lemma

**FB-Nest** A-Class, ind hyp

### B.1.13 Subtyping preserves absolute types

$\Gamma \vdash T <: T', \Gamma \vdash T \uparrow A \Rightarrow \Gamma \vdash T' \uparrow A'$

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** previous lemma, A-Class

**S-Nest-Class** trivial

**S-Var** def A-Var

**S-Out-Nest** A-Out

**S-Out-2** def A-Out, def A-Class

**S-Import** wf types lemma, previous lemma, wf prog, type substitution lemma

### B.1.14 Subtyping and inheritance of absolute types

$\Gamma \vdash T <: T', \Gamma \vdash T \uparrow A, \Gamma \vdash T' \uparrow A' \Rightarrow \vdash A \sqsubseteq_i^* A'$

Proof by induction on  $\Gamma \vdash T <: T'$  using previous lemma

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** A-Class, I-SC

**S-Nest-Class** ind hyp, inverting A-Class, FB-Nest, I-FB

**S-Var** by inversion of A-Var

**S-Out-Nest** ind hyp, either by inverting A-Out or applying A-Class for  $T$  and  $T'$  if a path or non-path type, respectively; lemma B.1.4.

**S-Out-2** inverting A-Class, A-Out, lemma B.1.8, reflexivity

**S-Import** SC-Import, lemma B.1.8, I-SC, transitivity

Corollary (Subtyping of absolute types gives inheritance):

$\Gamma \vdash A <: A' \Rightarrow \vdash A \sqsubseteq_i^* A'$

### B.1.15 Substitution (absolute types and well-formed types)

Let  $\Gamma = \Gamma_1, \overline{x:T}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\overline{s/x}] \Gamma_2$  where  $fv(\Gamma_1) \cap \overline{x} = \emptyset$  and  $\Gamma_1 \vdash s : [\overline{s/x}] T$ .

1. Absolute Types:  $\Gamma \vdash T \uparrow A, \Gamma_1 \vdash \overline{s} \uparrow \overline{A} \Rightarrow \Gamma' \vdash [\overline{s/x}] T \uparrow A' \vdash A' \sqsubseteq_i^* [\overline{s/x}] A$ . Proof by s. ind., case analysis on last step:

**A-Var** By cases on  $s(\Gamma(s) = T')$ :

$s \in \text{dom}(\Gamma_1) \quad [\overline{s/x}] s = s, \Gamma'(s) = \Gamma(s)$

$s = \mathbf{x}_i \quad [\overline{s/x}] T = s_i$ . premise of A-Var:  $\Gamma \vdash T_i \uparrow A$ , inversion lemma ('variables'), weakening:  $\Gamma \vdash s_i <: T_i$ , premise:  $\Gamma \vdash \varrho_i \uparrow A_i$ , lemma B.1.14:  $\vdash A_i \sqsubseteq_i^* A$ .  $A' = A_i$  by premise  $\Gamma \vdash \overline{s} \uparrow \overline{A}$  and lemma B.1.8;  $A = [\overline{s/x}] A$

$s \in \text{dom}(\Gamma_2) \quad [\overline{s/x}] s = s, \Gamma'(s) = [\overline{s/x}] \Gamma(s)$ , ind hyp

**A-World** trivial

**A-Out** ind hyp, lemma B.1.4

**A-Class** ind hyp, transitivity, FB-Nest

2. Well-formedness:  $\Gamma \vdash T \text{ OK}, \Gamma_1 \vdash \overline{s \text{ OK}} \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]T \text{ OK}$ . Proof follows from above part of lemma.

### B.1.16 (

Well-formed types)

1. Absolute Types:  $\Gamma \vdash T \uparrow A \Rightarrow \Gamma \vdash A \text{ OK}$  prof by s. ind.

**A-Var** ind hyp

**A-World** trivial

**A-Out** ind hyp, inverting A-Class

**A-Class** ind hyp, A-Class

2. Field types:  $fType(\mathfrak{f}, A) = T \Rightarrow \mathbf{this}: A \vdash T \text{ OK}$  by T-Class, substitution with lemma B.1.11

3. Method types:  $mType(\mathfrak{m}, A) = \Pi \overline{x}: \overline{T}. T \Rightarrow \mathbf{this}: A, \overline{x}: \overline{T} \vdash \overline{T}, T \text{ OK}$  by T-Class, substitution with lemma B.1.11

4. Type checking:  $\Gamma \vdash e : T \Rightarrow \Gamma \vdash T \text{ OK}$  Proof by s. ind.

**T-Subs** trivial

**T-Field** by  $fType$  case, substitution

**T-Assign** ind hyp

**T-New** F-Type

**T-Null** trivial

**T-Var** trivial

**T-Invk** by  $mType$  case, substitution

**T-Let** trivial

### B.1.17 Absolute types gives subtyping (S-Abs)

$\Gamma \vdash T \uparrow A \Rightarrow \Gamma \vdash T <: A$

Proof by s.ind, with a case analysis:

**A-Var** ind hyp, S-Var

**A-World** trivial

**A-Out** ind hyp, reflexivity of abs-types, def F-Type, S-Out-Nest

**A-Class** ind hyp, S-Nest-Class

### B.1.18 Conformance of type checking and absolute types

$\Gamma \vdash s \uparrow A \Rightarrow \Gamma \vdash s : A$

case analysis:

**A-Var** F-Path-Type, T-Var, lemma well-formed from absolute types, S-Var, lemma S-Abs, S-Trans, T-Subs

**A-World** F-Path-Type, T-Var

### B.1.19 Conformance of type checking and subtyping

$\Gamma \vdash s : T \Rightarrow \Gamma \vdash s <: T$

Proof by s. ind., case analysis:

**T-Subs** ind hyp, wf types lemma, inversion of F-Path-Type, determinacy of abs type lemma

**T-Var** inversion of F-Path-Type, determinacy of abs type lemma

### B.1.20 Substitution (subtyping and type checking)

Let  $\Gamma = \Gamma_1, \overline{x}: \overline{T}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\overline{s/\overline{x}}]\Gamma_2$  where  $fv(\Gamma_1) \cap \overline{x} = \emptyset$  and  $\Gamma_1 \vdash \overline{s} : [\overline{s/\overline{x}}]\overline{T}$ .

1. Subtyping:  $\Gamma \vdash T_1 <: T_2, \Gamma_1 \vdash \overline{s \text{ OK}} \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]T_1 <: [\overline{s/\overline{x}}]T_2$ . Proof by s. ind., case analysis on last step:

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** absolute types case, SC-Nest

**S-Nest-Class** ind hyp, well-formedness case

**S-Var** By cases on  $\gamma$ :

$\gamma \in \text{dom}(\Gamma_1) \quad [\overline{s/\overline{x}}]T_1 = T_1, \Gamma'(T_1) = \Gamma(T_1)$ , S-Var

$\gamma = \mathbf{x}_i \quad [\overline{s/\overline{x}}]T_1 = \varrho_i, T_2 = T_i$ , done by conformance of typing/subtyping lemma, weakening

$\gamma \in \text{dom}(\Gamma_2) \quad [\overline{s/\overline{x}}]T_1 = T_1$ , S-Var,  $\Gamma'(T_1) = [\overline{s/\overline{x}}]\Gamma(T_1)$

**S-Out-Nest** ind hyp, abs type case, lemma B.1.7

**S-Out-2** absolute types case, lemma B.1.7, lemma B.1.4, A-Out, S-SubClass, transitivity

**S-Import** abs types case, transitivity

2.  $fType$ :  $fType(\mathfrak{f}, A) = T \Rightarrow fType(\mathfrak{f}, [\overline{s/\overline{x}}]A) = [\overline{s/\overline{x}}]T$  proof by def class lookup, inheritance,  $fType$

3.  $mType$ :  $mType(\mathfrak{m}, A) = \Pi \overline{x}: \overline{T}. T \Rightarrow mType(\mathfrak{m}, [\overline{s/\overline{x}}]A) = [\overline{s/\overline{x}}]\Pi \overline{x}: \overline{T}. T$  proof by def class lookup, inheritance,  $fType$

4. Type checking:  $\Gamma \vdash e : T, \Gamma_1 \vdash \overline{s \text{ OK}} \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]e : [\overline{s/\overline{x}}]T$ . Proof by s. ind., case analysis on last step:

**T-Subs** ind hyp, subtyping case, well-formedness case

**T-Field** absolute types case,  $fType$  case, lemma B.1.7, well-form field lemma

**T-Assign** absolute types case,  $fType$  case, lemma B.1.7, ind hyp, well-form field lemma

**T-New** absolute types case, lemma B.1.7, ind hyp, well-form field lemma

**T-Null** well-formedness case

**T-Var** well-formedness case

**T-Invk** absolute types case,  $mType$  case, lemma B.1.7, ind hyp, well-form method lemma

**T-Let** ind hyp, well-formedness case

### B.1.21 *mType* and *mBody* are related

$mType(m, A) = \Pi \bar{x}:T.T, mBody(m, A) = (\bar{x}; e) \Rightarrow$   
**this**:  $A, \bar{x}:T \vdash e : T$  Proof by inversion of *mBodydef*, lemma B.1.7, T-Prog, T-Class, T-Method, substitution lemma with lemma B.1.11, type substitution lemma, (lemma B.1.9, invert F-Path-Type, reflexivity of abs types lemma, A-Class), T-Subs.

### B.1.22 Reduction preserves heap judgements

- $\mathcal{H} \vdash T \uparrow A, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}' \Rightarrow \mathcal{H}' \vdash T \uparrow A$
- $\mathcal{H} \vdash T \text{ OK}, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}' \Rightarrow \mathcal{H}' \vdash T \text{ OK}$
- $\mathcal{H} \vdash T <: T', e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}' \Rightarrow \mathcal{H}' \vdash T <: T'$
- $\mathcal{H} \vdash e_0 : T, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}' \Rightarrow \mathcal{H}' \vdash e_0 : T$

Proof by induction on  $e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}'$ , no rule modifies types in the heap, so  $\mathcal{H}^\Gamma = \mathcal{H}'^\Gamma$ , except for R-New and R-Let, which are by weakening.

### B.1.23 Preservation

$\vdash \mathcal{H} \text{ OK}, \mathcal{H} \vdash e : T, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}', e' \neq \text{err} \Rightarrow \vdash \mathcal{H}' \text{ OK}, e' = v, \mathcal{H}' \vdash v : T$

Proof by s. induction on derivation of  $e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}'$ , case analysis on last step:

**R-Field** inversion lemma, ind hyp, F-Heap

**R-New** for  $\mathcal{H}' \vdash v : T$ : inversion lemma, ind hyp, lemma B.1.22, T-Var, S-Var, weakening, transitivity. For  $\vdash \mathcal{H}' \text{ OK}$ : inversion lemma, well-formed expressions lemma, premises or *fTypecase*, substitution and T-Null.

**R-Assign** for  $\mathcal{H}' \vdash v : T$ : inversion lemma, ind hyp, well-formed types, transitivity, lemma B.1.22, T-Subs. For  $\vdash \mathcal{H}' \text{ OK}$ : inverting A-Var, ind hyp, lemma B.1.22.

**R-Invoke** inversion lemma, ind hyp, lemma B.1.22, lemma B.1.18, lemma B.1.7 and lemma B.1.21, substitution with weakening, transitivity, T-Subs

**R-Let** inversion lemma, ind hyp, substitution lemma,

All other cases are trivial.

### B.1.24 Subtyping preserves ownership

- Absolute types and out substitution:  $\Gamma \vdash P \uparrow A, fType(\text{out}, A) \text{ defined} \Rightarrow \Gamma \vdash [P/x]x.\text{out} \uparrow A'$ . Proof by case analysis on syntax of  $P$ :  
 $P = p [P/x]x.\text{out} = p.\text{out}$  by A-Out  
 $P = P'.N [P/x]x.\text{out} = P'$  by inverting A-Class  
Corollary: owners are well-formed:  $\Gamma \vdash P \Downarrow P' \Rightarrow \Gamma \vdash P' \uparrow A$ , proof by inverting O-Owner and above lemma.
- Subtyping increases precision:  $\Gamma \vdash T <: p \Rightarrow T = p'$  and  $\Gamma \vdash T <: \gamma.N \Rightarrow T \in \{p, p', N'\}$ . Proof by s. ind. on  $\Gamma \vdash T <: T'$ , with a case analysis on the last step:

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** trivial

**S-Nest-Class** ind hyp

**S-Var** trivial

**S-Out-Nest** by cases on  $P'$ , def subst, ind hyp

**S-Out-2** trivial

**S-Import** N/A

- Subtyping preserves ownership:  $\Gamma \vdash P <: P', \Gamma \vdash P' \Downarrow P'_0 \Rightarrow \Gamma \vdash P \Downarrow P_0, \Gamma \vdash P_0 <: P'_0$ . Proof by s. induction on the derivation of the subtyping premise, case analysis on the last step:

**S-Reflex** trivial

**S-Trans** ind hyp

**S-SubClass** easy, by def O-Owner, def subst

**S-Nest-Class** by inverting A-Class, lemma B.1.4, we get  $\vdash P \sqsubseteq_i^* P'$  (referring to  $P$  from the rule), by inverting O-Owner and def *fType* we get the owner of  $P_0$ , subtyping is given by premise of rule

**S-Var** inverting O-Owner, A-Var, S-Out-Nest, O-Owner

**S-Out-Nest** invert O-Owner, use lemma B.1.14, def of *fType*, and first lemma to get  $\Gamma \vdash P \uparrow A$ , then lemma B.1.14 and def *fType* again, with O-Owner to get  $P_0$ , finally S-Out-Nest to get subtype relationship.

**S-Out-2** inverting O-Owner, def subst on paths, inverting A-Out, applying O-Owner

**S-Import** N/A — since rhs must not have **out** field by wf-prog, T-Class, but **out** must be defined for a type to have an owner by O-Owner.

## B.2 Semantic o-as-local-d

### B.2.1 Naming and types

$\Gamma \vdash P \oplus T, \Gamma(x) = P \Rightarrow \Gamma \vdash x \oplus T$ .

Proof by s. induction on  $\Gamma \vdash P \oplus T$ :

**N-O-As-D** S-Var, def A-Var, I-Sub, I-Trans

**N-Public** ind hyp

**N-Type-Var** trivial

### B.2.2 Type substitution (inside, owners, naming)

Let  $\Gamma = \Gamma_1, \bar{x}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\bar{T}/\bar{X}]\Gamma_2$  where  $\vdash \Gamma_1 \text{ OK}$  and  $fV(\Gamma_1) \cap \bar{x} = \emptyset$  and  $\Gamma_1 \vdash \bar{T} \text{ OK}$ .

- Inside relation:  $\Gamma \vdash P \prec: P' \Rightarrow \Gamma' \vdash [\bar{T}/\bar{X}]P \prec: [\bar{T}/\bar{X}]P'$ ; proof by s. induction over derivation, etc.:

**I-Sub** subtyping case, abs type case, def *public*

**I-Trans** ind hyp

**I-Out** trivial

**I-Class** trivial

**I-Runtime** trivial

2. Owner relation:  $\Gamma \vdash P \Downarrow P' \Rightarrow \Gamma' \vdash [\overline{T/\overline{x}}]P \Downarrow [\overline{T/\overline{x}}]P'$ ; proof by abs type case, def *fType*, def subst.
3. Naming:  $\Gamma \vdash \mathbf{x} \oplus T, \Gamma_1 \vdash \Gamma'(\mathbf{x}) \oplus \overline{\mathbf{T}} \Rightarrow \Gamma' \vdash \mathbf{x} \oplus [\overline{T/\overline{x}}]T$ ; proof by s. induction over derivation, etc.:

**N-O-As-D** owner case, inside case

**N-Public** abs type case, def *public*, ind hyp

**N-Type-Var** lemma B.2.1, second premise

### B.2.3 Substitution (inside, owners, naming)

Let  $\Gamma = \Gamma_1, \overline{\mathbf{x}:T}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\overline{s/\overline{x}}]\Gamma_2$  where  $\vdash \Gamma_1$  OK and  $fv(\Gamma_1) \cap \overline{\mathbf{x}} = \emptyset$  and  $\Gamma_1 \vdash s : [\overline{s/\overline{x}}]T$ .

1. Inside relation:  $\Gamma \vdash P \prec: P' \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]P \prec: [\overline{s/\overline{x}}]P'$ ; proof by s. induction over derivation, etc.:

**I-Sub** subtyping case, abs type case, def *public*

**I-Trans** ind hyp

**I-Out** trivial

**I-Class** trivial

**I-Runtime** easy since  $\iota \notin \overline{\mathbf{x}}$

2. Owner relation:  $\Gamma \vdash P \Downarrow P' \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]P \Downarrow [\overline{s/\overline{x}}]P'$ ; proof by abs type case, def *fType*, def subst.

3. Naming:  $\Gamma \vdash P \oplus T \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]P \oplus [\overline{s/\overline{x}}]T$ ; proof by s. induction over derivation, etc.:

**N-O-As-D** owner case, inside case

**N-Public** abs type case, def *public*, ind hyp

**N-Type-Var** trivial

### B.2.4 Owners are local dominators

Define *oasld*( $\mathcal{H}$ ) if  $\forall \iota, \iota' \in \mathcal{H}$  where  $\mathcal{H} \vdash \iota \hookrightarrow \iota'$ .  $\Gamma \vdash \iota \prec: \iota'.$  **out**  $\vee (\mathcal{H} \vdash \mathcal{H}(\iota') \uparrow A \wedge \text{public}(A))$

Where  $\hookrightarrow$  means has a reference to.

$\vdash \mathcal{H}$  OK,  $\mathcal{H} \vdash e : T, e; \mathcal{H} \rightsquigarrow e'; \mathcal{H}', \text{oasld}(\mathcal{H}) \Rightarrow \text{oasld}(\mathcal{H}')$

Proved as part of preservation by extra premises of F-Heap. Proof obligation is to prove the extra premises of F-Heap for the result heap of a reduction:

**R-New** ind hyp, inversion lemma, ind hyp (preservation), lemma B.1.18, lemma B.1.24, def *owner* and *subst* (gives  $\mathcal{H}_n \vdash \overline{p} \prec: r.$  **out** where  $\mathcal{H}_n \vdash [\iota/\mathbf{this}]T \Downarrow \overline{p}$ ); by def A-Class, wf-prog, T-Class:  $\Gamma \vdash \mathbf{this} \oplus \overline{\mathbf{T}}$ , where  $\Gamma = \mathbf{this}:C<\overline{\mathbf{x}}>, \overline{\mathbf{Y}}, \overline{\mathbf{x}}$ , by type *subst* (naming), *subst* (naming), weakening:  $\mathcal{H}_n \vdash \iota \oplus [\iota/\mathbf{this}]T$ . Done by def naming rules and transitivity of inside relation.

**R-Let** ind hyp

**R-Assign** the interesting part of the argument follows R-New.

**R-Invoke** ind hyp

All other cases are trivial

## B.3 Syntactic O-as-local-d

Omitted lemmas are identical to semantic version of naming

### B.3.1 Naming and types

$\Gamma \vdash P \oplus T, \Gamma(\mathbf{x}) = P \Rightarrow \Gamma \vdash \mathbf{x} \oplus T$ .

Proof by S-Var, N-Sub.

### B.3.2 Type substitution (naming)

Let  $\Gamma = \Gamma_1, \overline{\mathbf{x}:T}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\overline{T/\overline{x}}]\Gamma_2$  where  $\vdash \Gamma_1$  OK and  $fv(\Gamma_1) \cap \overline{\mathbf{x}} = \emptyset$  and  $\Gamma_1 \vdash \overline{\mathbf{T}}$  OK.

Naming:  $\Gamma \vdash \mathbf{x} \oplus T, \Gamma_1 \vdash \Gamma'(\mathbf{x}) \oplus \overline{\mathbf{T}} \Rightarrow \Gamma' \vdash \mathbf{x} \oplus [\overline{T/\overline{x}}]T$ ; proof by s. induction over derivation, etc.:

**N-Reflex** trivial

**N-Rep** trivial

**N-Public** abs type case, def *public*, ind hyp

**N-Type-Var** lemma B.3.1, second premise

**N-Out-1** ind hyp

**N-Out-2** trivial

**N-Sub** ind hyp, subtype case

### B.3.3 Substitution (naming)

Let  $\Gamma = \Gamma_1, \overline{\mathbf{x}:T}, \Gamma_2$  and  $\Gamma' = \Gamma_1, [\overline{s/\overline{x}}]\Gamma_2$  where  $\vdash \Gamma_1$  OK and  $fv(\Gamma_1) \cap \overline{\mathbf{x}} = \emptyset$  and  $\Gamma_1 \vdash s : [\overline{s/\overline{x}}]T$ .

Naming:  $\Gamma \vdash P \oplus T \Rightarrow \Gamma' \vdash [\overline{s/\overline{x}}]P \oplus [\overline{s/\overline{x}}]T$ ; proof by s. induction over derivation, etc.:

**N-Reflex** trivial

**N-Rep** trivial

**N-Public** abs type case, def *public*, ind hyp

**N-Type-Var** trivial

**N-Out-1** ind hyp

**N-Out-2** trivial

**N-Sub** ind hyp, subtype case

### B.3.4 Syntactic name restriction gives semantic name restriction

$\vdash \Gamma$  OK,  $\Gamma \vdash P \oplus T(\text{syntactic}) \Rightarrow \Gamma \vdash P \oplus T(\text{semantic})$ .

Proof by s.ind on the syntactic derivation:

**N-Reflex** S-Reflex, I-Sub, N-O-As-D

**N-Rep** def O-Owner, S-Reflex, I-Sub, N-O-As-D

**N-Public** trivial

**N-Type-Var** trivial

**N-Out-1** ind hyp, *rhs = p* means we can invert N-O-As-D, def O-Owner; in the second conclusion we are done, in the first, then by I-Out

**N-Out-2** def O-Owner, I-Out, N-O-As-D

**N-Sub** ind hyp, cases analysis on last semantic rule applied:  
 N-Type-Var case is trivial; N-Public: by lemma B.1.13 and lemma B.1.14, def *public*, N-Public; N-O-As-D: by I-Out, I-Trans.

### B.3.5 Owners are local dominators

Identical to semantic case, but using the lemmas defined for syntactic name restriction and lemma B.3.4

## C. Effects

### Effects syntax

$$\varphi ::= \varphi, P \mid \varphi, @P \mid \emptyset \quad \text{effects}$$

### Expression effects $\boxed{\Gamma \vdash e! \varphi}$

$\frac{\Gamma \vdash e : \varphi; \varphi' \quad \Gamma \vdash \varphi \subseteq \varphi'' \quad \Gamma \vdash \varphi' \subseteq \varphi''' \quad \Gamma \vdash \varphi''; \varphi''' \text{ OK}}{\Gamma \vdash e! \varphi''; \varphi'''}$ <p style="text-align: center;">(E-SUBS)</p>	$\frac{\Gamma, \mathbf{x}:T \vdash e' : \varphi; \varphi' \quad \Gamma \vdash \varphi; \varphi' \text{ OK}}{\Gamma \vdash \mathbf{let} \ \mathbf{x}:T = e \ \mathbf{in} \ e'! \varphi; \varphi'}$ <p style="text-align: center;">(E-LET)</p>
$\frac{}{\Gamma \vdash \gamma. \mathbf{f}! @\gamma; \emptyset}$ <p style="text-align: center;">(E-FIELD)</p>	$\frac{\Gamma \vdash e! \varphi; \varphi'}{\Gamma \vdash \gamma. \mathbf{f} = e! \varphi, @\gamma; \varphi', @\gamma}$ <p style="text-align: center;">(E-ASSIGN)</p>
$\frac{}{\Gamma \vdash \mathbf{null}! \emptyset; \emptyset}$ <p style="text-align: center;">(E-NULL)</p>	$\frac{\Gamma \vdash s \text{ OK}}{\Gamma \vdash s! \emptyset; \emptyset}$ <p style="text-align: center;">(E-VAR)</p>
$\frac{\Gamma \vdash \overline{e : \varphi; \varphi'}}{\Gamma \vdash \mathbf{new} \ \gamma. N(\overline{\mathbf{ff}}=e)! \overline{\varphi}; \varphi'}$ <p style="text-align: center;">(E-NEW)</p>	
$\frac{\Gamma \vdash \gamma \uparrow A \quad mEffect(m, A) = \varphi; \varphi'}{\Gamma \vdash \gamma. m(\overline{\mathbf{x}})! \varphi, @\gamma; \varphi'}$ <p style="text-align: center;">(E-INVK)</p>	

**Figure 18.**  $\text{Tribe}_{own}$  effect checking rules rules.

We present the syntax of effects and rules for effect checking in Fig. 18, and sub-effecting in Fig. 19. We assume there is an effect annotation on methods and a lookup function  $mEffect$  that does the obvious lookup (for soundness we must assume effects in a subclass are a sub-effect of the same method in the superclass).  $@T$  effects mean only those objects owned by  $T$  or  $T$  (if  $T$  is a singleton type) are read/written.  $T$  effects denote an effect on  $T$  or objects transitively owned by  $T$ . If  $T$  is not a singleton type, then it describes many contexts. The usual disjointness rules should apply [18].

### Sub-effects $\boxed{\Gamma \vdash \varphi \subseteq \varphi'}$

$\frac{}{\Gamma \vdash \varphi \subseteq \varphi}$ <p style="text-align: center;">(SE-REFLEX)</p>	$\frac{\Gamma \vdash \varphi \subseteq \varphi'' \quad \Gamma \vdash \varphi'' \subseteq \varphi'}{\Gamma \vdash \varphi \subseteq \varphi'}$ <p style="text-align: center;">(SE-TRANS)</p>
$\frac{}{\Gamma \vdash @T \subseteq T}$ <p style="text-align: center;">(SE-EXACT)</p>	$\frac{\Gamma \vdash \varphi_2 \subseteq \varphi'_2}{\Gamma \vdash \varphi_1, \varphi_2, \varphi_3 \subseteq \varphi_1, \varphi'_2, \varphi_3}$ <p style="text-align: center;">(SE-SEQ)</p>
$\frac{\Gamma \vdash T <: T' \quad \Gamma \vdash T \subseteq T'}{\Gamma \vdash @T \subseteq @T'}$ <p style="text-align: center;">(SE-SUBTYPE)</p>	$\frac{\Gamma \vdash T <: T'}{\Gamma \vdash T \subseteq T'}$ <p style="text-align: center;">(SE-INSIDE)</p>
$\frac{\Gamma \vdash \varphi_1 \subseteq \varphi_3 \quad \Gamma \vdash \varphi_2 \subseteq \varphi_3}{\Gamma \vdash \varphi_1, \varphi_2 \subseteq \varphi_3}$ <p style="text-align: center;">(SE-ELIM)</p>	$\frac{}{\Gamma \vdash \varphi_2 \subseteq \varphi_1, \varphi_2, \varphi_3}$ <p style="text-align: center;">(SE-INTRO)</p>

**Figure 19.** Enforcing encapsulation properties in  $\text{Tribe}_{own}$ .