TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

EXAMINATIONS – 2017, TRIMESTER 1

Surname .....

First Name .....

Student Number .....

ENGR 121 ENGINEERING MATHEMATICS FOUNDATIONS

Time Allowed: THREE HOURS

CLOSED BOOK

Permitted materials: Silent non-programmable calculators or silent programmable calculators with their memories cleared are permitted.
Printed foreign language dictionaries are permitted.
A formula sheet is provided separately.
No other material is permitted.

Instructions: Answer all 8 questions. The exam will be marked out of a total of 100.
Answer in the appropriate boxes if possible — if you write your answer elsewhere, make it clear where your answer can be found. Please use the blank reverse sides of pages and/or the 2 blank end-pages for any extra space you need, for working or for answers.

Questions	Marks
1. Set Theory	[10]
2. Functions & Relations	[20]
3. Logic	[10]
4. Probability	[10]
5. Series	[10]
6. Differentiation	[15]
7. Integration	[10]
8. Vectors & Matrices	[15]

For marking use only

1	
2	
3	
4	
5	
6	
7	
8	
Total	

Student Number .....

1. Set Theory **(10 marks)**

(a) **(5 marks)** State whether each of the following is true or false:

$-2.13 \in \mathbb{Q}$	$\sqrt{2} \in \mathbb{Q}$	$\mathbb{N} \subset \mathbb{Z}$
$-413 \in \mathbb{Z}$	$\mathbb{R} \subset \mathbb{Q}$	

(b) **(5 marks)** Simplify where possible the following operations on sets:

$A \cup \overline{A}$	$\overline{\overline{\overline{A}}}$	$A \cap \mathbb{E}$
$\overline{A} \cup \phi$	$A \cap (A \cup B)$	

/ 10

2. Functions and Relations **(20 marks)**

(a) **(3 marks)** Sketch a graph of the two functions $f(x) = 2 + x$ and $g(x) = 3 - 2x$, on the same axes. Solve $f(x) = g(x)$ and use your solution to mark on the graph the region where $g(x) > f(x)$.

/ 3

(b) **(3 marks)** Is the function $f(x) = \dfrac{1}{1+x^2}$ one-to-one, one-to-many, or many-to-one?

What is the domain of $f(x)$? What is the range of $f(x)$? What is the range of $2f(x) - 5$?

(c) **(3 marks)** Consider the power function, $f(x) = x^r$, for some real number, r. Sketch a graph of the function $f(x)$ for $r = -1$, $r = 0$, $r = 1$ and $r = 2$, on the same axes.

(d) **(3 marks)** Show that $x = 2$ is a root of the equation $x^3 - 3x^2 + 4 = 0$. Find the other roots.

(e) **(1 mark)** Solve $\sqrt{e^{2y}} = 4$.

(f) **(1 mark)** Simplify the following expression as much as possible: $(w^2)^3 - log(w^3) + \log(e^{-3w})$.

/ 11

(g) **(1 mark)** The response of a system is given by a signal of amplitude: $A(x) = \dfrac{6}{2 + (\frac{x-3}{4})^2}$. What value of x maximizes the amplitude?

(h) **(2 marks)** Find all solutions of the equation $\cos(x) = 0.3$, for $x \in [0, 2\pi]$.

(i) **(3 marks)** Consider the two functions, $f(x) = (x-1)^2$ and $g(x) = ax + 2$ for some real number, a. Write down the function $f(g(x))$. What value of a gives $x = 10$ as a root of the equation, $f(g(x)) = 0$?

/ 6

3. Logic **(10 marks)**

(a) **(2 marks)** Prove that $A + \overline{B} = A + (\overline{A} \cdot \overline{B})$ using truth tables.

(b) **(2 marks)** Draw a circuit diagram for the expression: $A \cdot B + \overline{C}$

/ 4

(c) **(2 marks)** Simplify where possible the expression: $A \cdot (A + B) + \bar{B} \cdot A$.

(d) **(2 marks)** Write the disjunctive normal form for a boolean expression with inputs A, B, and C, and with output X, that has the truth table

A	B	C	X
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

(e) **(2 marks)** Simplify the disjunctive normal form as much as you can.

/ 6

4. Probability **(10 marks)**

(a) **(5 marks)** Given $P(A \cap B) = 0.2$, $P(A \cap \overline{B}) = 0.3$, $P(\overline{A} \cap B) = 0.1$ and $P(\overline{A} \cap \overline{B}) = 0.4$, answer the following questions:

(i) Find $P(A)$.

(ii) Find $P(\overline{B})$.

(iii) Find $P(A \cup B)$.

(iv) Find $P(A|B)$.

(v) Are A and B independent? Give reasons for your answer.

/ 5

(b) **(5 marks)** Defects in the production of a new experimental material are detected by a scanner. Defects occur 10% of the time. The scanner successfully detects a defect 80% of the time. When there is no defect, the scanner reports there is no defect 90% of the time.

(i) Draw a tree diagram for this situation. Include labels for the nodes in your diagram and label the arcs with the relevant probabilities. Also record the overall probabilities on the right hand side of the tree corresponding to each outcome.

(ii) What is the probability the scanner indicates a defect?

(iii) What is the probability a defect is present if the scanner indicates a defect?

(iv) What is the probability a defect is indicated by the scanner if there is no defect?

(v) Is it possible for two probabilities of the form $P(A|B)$ and $P(A)$ to satisfy $P(A|B) < P(A)$? If it is possible, give an example by providing valid values of $P(A)$, $P(B)$ and $P(A|B)$.

/ 5

5. Series **(10 marks)**

(a) **(2 marks)** Write down the first three terms, then sum the arithmetic series $\sum_{i=0}^{10}(1+3i)$.

(b) **(2 marks)** Evaluate the geometric series $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$.

(c) **(2 marks)** By considering the power series expansion of e^x, find an exact value for the infinite series $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k!}$

(d) **(2 marks)** Using the power series expansion for $\cos x$, write down the first three terms of the power series expansion for $\cos(2x)$

(e) **(2 marks)** Use the extended binomial theorem to write down the first three terms of the power series for $\sqrt{1-2x}$.

/ 10

6. Differentiation **(15 marks)**

(a) **(2 marks)** Find the derivative of $f(t) = t^2 + t$ from first principles, that is, from the definition of the derivative.

(b) **(1 mark)** A function $y(x)$ is such that dy/dx is equal to y for all values of x. What can you say about $y(x)$?

(c) **(4 marks)** Find the derivatives y' for the following four functions. You may use the table of derivatives provided in the formula sheet.

(i) $y = 3/x^3$

(ii) $y = \pi^2 x$

/ 5

(iii) $y = \sin(3 - x)$

(iv) $y = (\ln(5x))^{-2}$

(d) **(5 marks)** Differentiate the five functions

(i) $y = (x^2 + 1)\sin x$

(ii) $y = x^3 \ln x$

(iii) $y = \dfrac{\cos(2t)}{\sin(3t)}$

(iv) $y = (1 + 3t^2)^{-13}$

/ 6

(v) $y = e^{-x^3}$

(e) **(1 mark)** Calculate d^2y/dt^2, given that $y = t^3 + 5t$.

(f) **(2 marks)** Find all local maxima and minima of the function

$$y = x(x^2 - 6x + 9)$$

Using the second derivative, show which are minima and which are maxima.

/ 4

7. Integration **(10 marks)**

(a) **(5 marks)** Find the following integrals

(i) $\int (x^2 + 5)\,\mathrm{d}x$

(ii) $\int \cos(5x)\,\mathrm{d}x$

(iii) $\int -7x^{-5}\,\mathrm{d}x$

(iv) $\int \frac{\pi}{x}\ dx$

(v) $\int 6e^{3x}\,\mathrm{d}x$

/ 5

(b) **(2 marks)** Evaluate the integral

$$\int_0^1 x\, e^{3x}\, \mathrm{d}x$$

using *integration by parts*

$$\int u \left(\frac{\mathrm{d}v}{\mathrm{d}x}\right) \mathrm{d}x \;=\; uv - \int \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right) v\, \mathrm{d}x$$

(c) **(3 marks)** Perform an integration to determine the *average value* of the function

$$f(t) \;=\; 1 - 3t^2$$

over the interval $0 \leq t \leq 1$.

/ 5

8. Vectors and Matrices **(15 marks)**

(a) **(4 marks)**

Given

$$\boldsymbol{a} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \qquad \boldsymbol{c} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Find

(i) $\boldsymbol{a} - 2\boldsymbol{b} + \boldsymbol{c}$

(ii) $|\boldsymbol{b} + 2\boldsymbol{c}|$

(iii) $\boldsymbol{c} \cdot \boldsymbol{b}$

(iv) the angle θ between $\boldsymbol{c}$ and $\boldsymbol{b}$

/ 4

(b) **(4 marks)** Consider the points $P(1,-2,1)$, $Q(-1,1,1)$ and $R(-1,-1,-2)$ in $\mathbb{R}^3$.

(i) Determine the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(ii) Write down a vector equation for the line through P and Q.

(iii) Using the vectors computed in (i), compute a vector which is perpendicular to the plane containing P, Q and R.

(iv) Write down a vector equation for the plane containing P, Q and R.

/ 4

(c) **(4 marks)** Given

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(i) Find $A - B$

(ii) Find AB

(iii) Find B^{-1}

(iv) Calculate $det(A)$ and $det(B)$.

/ 4

(d) **(3 marks)**
(i) Write down a 2x2 transformation matrix A which will reflect an object in the x-axis.

(ii) Write down a 2x2 transformation matrix B which will reflect an object in the y-axis and double its height and its width.

(iii) Using matrix multiplication, check that your transformation matrices A and B have the desired effect on the unit square.

/ 3

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

* * * * * * * * * * * * * * *