TEST + SOLS

ENGR121 Test 2 2016

Student Name: ....

Question 1. Differentiation [9 marks]

Find the derivatives y' for the following functions. You may use the table of derivatives provided in the formula sheet.

(a) $y = -3x^{-3}$

$9x^{-4}$

(b) $y = e^{-2}$

0

(c) $y = 10e^{-5x+3}$

$10(-5)e^{-5x+3} = -50e^{-5x+3}$

(d) $y = \cos(kx)$

$-k\sin(kx)$

(e) $y = \sin x \ \ln(3x)$

$\sin x \frac{3}{3x} + \cos x \ln(3x) = \frac{\sin x}{x} + \cos x \ln(3x)$

(f) $y = e^x \cos x$

$e^x(-\sin x) + e^x \cos x = e^x(\cos x - \sin x)$

(g) $y = (\ln t)/t^2$

$\dfrac{\frac{1}{t} t^2 - \ln(t) 2t}{(t^2)^2} = \dfrac{t - 2t \ln t}{t^4}$

(h) $y = (3t^4 + 4t)^{-40}$

$-40(12t^3 + 4)(3t^4 + 4t)^{-41}$

(i) Harder-have some fun with this one!

$y = \ln\left(e^{-\cos(3x)}\right)$

$y' = \dfrac{1}{e^{-\cos(3x)}} 3\sin(3x) e^{-\cos 3x}$

$= 3\sin(3x)$ easy if you spot

$y = -\cos(3x)$!

Question 2. Integration [7 marks]

Find the following integrals

(a) $\int (x^{-2} - 7x^4)\,\mathrm{d}x$

$-x^{-1} - \frac{7x^5}{5} + c$

(b) $\int \cos\left(\frac{1}{3}x\right)\,\mathrm{d}x$

$3\sin\left(\frac{1}{3}x\right) + c$

(c) $\int \frac{-16}{x^6}\,\mathrm{d}x$

$\int -16x^{-6}\,dx \quad = \quad \frac{-16}{-5}x^{-5} + c$

(d) $\int \left(\frac{5}{x} + \frac{x}{5}\right)\,\mathrm{d}x$

$5\ln(x) + \frac{x^2}{10} + c$

(e) $\int_0^2 e^{-3x/2}\,\mathrm{d}x$

$-\frac{2}{3}e^{-3x/2} + c$

(f) The classic integration by parts: Find the following integral

$$\int x^2 e^x \, \mathrm{d}x$$

$$x^2 e^x - \int 2x e^x \, dx$$

$$x^2 e^x - \left[2x e^x - \int 2e^x \, dx \right]$$

$$x^2 e^x - 2x e^x + 2 \int e^x \, dx$$

$$x^2 e^x - 2x e^x + 2e^x + C$$

$$(x^2 - 2x + 2) e^x + C$$

Question 3. Vectors

[9 marks]

Given

$$a = \begin{bmatrix} -6 \\ 2 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \quad c = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$$

Find

(a) $2a - 3b - c$

$$\begin{pmatrix} -12 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -22 \\ 4 \\ -5 \end{pmatrix}$$

(b) $|c + b|$

$$\left| \begin{matrix} 6 \\ 2 \\ 1 \end{matrix} \right| = \sqrt{36+4+1} = \sqrt{41}$$

(c) $b \cdot c$

$$8 - 3 - 2 = 3$$

(d) the angle θ between c and b

$$3 = \sqrt{4+1+4}\sqrt{16+9+1} \cos\theta$$

$$3 = \sqrt{9}\sqrt{26} \cos\theta$$

$$\frac{3}{3\sqrt{26}} = \cos\theta$$

$$0.196 = \cos\theta$$

$$\cos^{-1} 0.196 = \theta$$

$$\theta = 1.373 \text{ rad}$$

$$= 78.7^{\circ}$$

(e) Trickier: Let a, b and c represent 3 points in 3D space. Find a vector perpendicular to the plane containing these 3 points.

$$\underset{\sim}{u} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix}$$

$$\underset{\sim}{w} = \underset{\sim}{c} - \underset{\sim}{a} = \begin{bmatrix} 10 \\ 1 \\ -1 \end{bmatrix}$$

lie on plane

$$\underset{\sim}{u} \times \underset{\sim}{w} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} \times \begin{bmatrix} 10 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -(-28) \\ 38 \end{bmatrix} = \begin{bmatrix} 1 \\ 28 \\ 38 \end{bmatrix}$$

is perpendicular.

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Function $f(x)$	*Derivative* $f'(x)$	*Indefinite integral* $\int f(x)\,dx$
k (constant)	0	$kx + c$
x	1	$\frac{1}{2}x^2 + c$
x^2	$2x$	$\frac{1}{3}x^3 + c$
x^n where $n \neq -1$	nx^{n-1}	$\frac{1}{n+1}x^{n+1} + c$
$\frac{1}{x}$ $(= x^{-1})$	$-x^{-2}$ $(= -\frac{1}{x^2})$	$\ln\|x\| + c$
$\sin x$	$\cos x$	$-\cos x + c$
$\sin(ax+b)$	$a\cos(ax+b)$	$-\frac{1}{a}\cos(ax+b) + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\cos(ax+b)$	$-a\sin(ax+b)$	$\frac{1}{a}\sin(ax+b) + c$
$\operatorname{cosec}(ax+b)$	$-a\operatorname{cosec}(ax+b)\cot(ax+b)$	$\frac{1}{a}\ln(\operatorname{cosec}(ax+b) - \cot(ax+b)) + c$
$\sec(ax+b)$	$a\sec(ax+b)\tan(ax+b)$	$\frac{1}{a}\ln(\sec(ax+b) + \tan(ax+b)) + c$
e^x	e^x	$e^x + c$
e^{kx}	ke^{kx}	$\frac{1}{k}e^{kx} + c$
$\ln x$	$\frac{1}{x}$	$x\ln x - x + c$
$\ln(kx)$	$\frac{k}{x}$	$x\ln(kx) - x + c$

Integration by parts

$$\int u\left(\frac{dv}{dx}\right)dx = uv - \int\left(\frac{du}{dx}\right)v\,dx$$

Derivative Rules:

Product Rule: if $y = uv$, then $y' = u'v + uv'$

Quotient Rule: if $y = u/v$, then

$$y' = \frac{(vu' - uv')}{v^2}$$

Chain Rule: if $y = y(z)$ and $z = z(x)$, then

$$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$