ENGR121 Test Two During lecture, 26 May 2017

Surname:
First Name:
Student Number:

Please use the spaces provided in this test booklet next to the questions, to give your answers. Please show all working. You may use page five for rough working if you need more space, plus the reverse sides of all pages.

Attempt all questions on pages 2–4. The mark for each subquestion is given in square brackets, e.g. [1].

Silent programmable calculators may be used. A table of formulae is available to you for this test.

Page totals, for marking use only

Page	**Mark**	**max**
p.2		13
p.3		10
p.4		12
Total		**35**

1. **Functions**

(a) Write down the oblique asymptote of the rational function [1]

$$y(x) = x - 3 + \frac{7}{x-5}$$

(b) Where is the vertical asymptote of the above rational function? [1]

(c) Simplify $\frac{e^x}{3e^{5x}}$ [1]

(d) Evaluate exactly $\log_2 16$ [1]

(e) Simplify to a single log term [1]

$$-5\log y + \log(xy)$$

(f) Solve exactly $\log_{10}(t^3) = 3$ [1]

(g) Solve $e^{3-x} = 1$ [1]

(h) Solve $\sqrt{e^{2x}} = 3$ [1]

(i) Solve $e^x(e^x - 2) = -1$.
Hint: Substitute $w = e^x$ to get a quadratic in w. [1]

(j) Simplify $\cos^2\theta \tan\theta$ [1]

(k) Find two solutions to $\sin\theta = 0.5$ for $\theta \in [0, \pi]$. [1]

2. **Sequences and Series**

(a) If $x[k] = \frac{k-2}{k+1}$, find $\lim_{k\to\infty} x[k]$ if it exists. [1]

(b) If $x[k] = (-0.9)^k$, find $\lim_{k\to\infty} x[k]$ if it exists. [1]

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(c) Find the sum to six terms of the geometric series with first term 2 and common ratio 1/5. Also find the sum to infinity. [2]

(d) Using the power series expansion for $\cos x$, write down the first three terms of the power series expansion for $\cos(3x)$ [1]

(e) By considering the power series expansion of e^x, find an exact value for the infinite series $\sum_{k=1}^{\infty} \frac{-3}{k!}$ [2]

3. **Differentiation**

(a) Write down the most general value of y that solves the equation [1]

$$\frac{dy}{dx} = 0$$

$y = c$

(b) What can you say about the derivative of the function $y = |x|$ at the point $x = 2$? [1]

For $x > 0$ $|x| = x$
so $y' = 1$

(c) What can you say about the derivative of the function $y = |x|$ at the point $x = 0$? [1]

Nothing - doesn't exist

(d) Find the derivatives y' of these functions

i. $y = e^{-7x}$ [1]

$y' = -7e^{-7x}$

ii. $y = \sin\left(\frac{x}{2} - 1\right)$ [1]

$y' = \frac{1}{2}\cos\left(\frac{x}{2} - 1\right)$

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iii. $y = \tan^{-1}(x+1)$ [1]

$$y' = \frac{1}{1+(x+1)^2}$$

(e) Use the product rule to differentiate

i. $y = \sin t \sin(2t)$ [1]

$$y' = (\sin t)\, 2\cos(2t) + (\cos t)\sin(2t)$$

ii. $y = \ln z \sin z$ [1]

$$\ln(z)\cos(z) + \frac{1}{z}\sin(z)$$

iii. $y = 3\cosh x \sinh(x+1)$ [1]

No sinh/cosh in this year's formula sheet.

(f) Use the quotient rule to find the derivatives of

i. $y = \sin t/\cos t$ [1]

$$\frac{(\cos t)\cos t - (-\sin t)\sin t}{\cos^2 t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t}$$

$$= \frac{1}{\cos^2 t} = \sec^2 t$$

ii. $y = \ln x/\sin x$ [1]

$$\frac{\frac{1}{x}\sin x - \cos x \ln x}{\sin^2 x}$$

iii. $y = (1+e^{-t})/(1-e^t)$ [1]

$$\frac{-e^{-t}(1-e^{t}) + e^{t}(1+e^{-t})}{(1-e^{t})^2}$$

(g) Use the chain rule to find the derivatives of

i. $y = e^{t^2}$ [1]

$$2te^{t^2}$$

ii. $y = \sqrt{3t-1}$ [1]

$$\tfrac{1}{2}(3t-1)^{-1/2} \times 3$$

(h) Find all local maxima and minima of $y = te^{-t}$. [3]

$$y' = e^{-t} + (-e^{-t})t$$

$$= e^{-t}(1-t)$$

$y' = 0$ at $t = 1$

$$y'' = -e^{-t} - e^{-t}(1-t)$$

At $t=1$ $y'' = -e^{-1} = \ominus$

$\Rightarrow$ MAX

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************* END OF TEST *************