ENGR121 - Test 2 - 2018

Surname: ....

First Name: ....

Student Number: ....

Please use the spaces provided in this test booklet next to the questions, to give your answers. Please show all working. You may use the last page for rough working if you need more space. Attempt all questions. The marks for parts of questions are given in square brackets, e.g. [1]. Silent calculators may be used.

A table of formulae is provided, you can detach it if you wish.

Question totals, for marking use only

Question	Mark	max
Q. 1		10
Q. 2		10
Q. 3		5
Total		25

1. Differentiation **(10 marks)**

   Find the derivatives y' for the following functions. You may use the table of derivatives provided in the formula sheet.

   (a) $y = x^{-1/3}$

   $y' = -\frac{1}{3}x^{-4/3}$

   (b) $y = e^{-2x}$

   $y' = -2e^{-2x}$

   (c) $y = \pi^{-2}$

   $y' = 0$

   (d) $y = \cos x \ \ln(2x)$ (use product rule)

   $y' = (-\sin x)\ln(2x) + (\cos x)\frac{1}{x}$

   $= \frac{\cos x}{x} - \sin x \ln(2x)$

(e) $y = (2x^3 - x)\sin(2x)$ (use product rule)

$$y' = (6x^2 - 1)\sin(2x) + (2x^3 - x)2\cos(2x)$$

(f) $y = e^t / \sin t$ (use quotient rule)

$$y' = \frac{e^t \sin t - (\cos t)e^t}{\sin^2 t}$$

(g) $y = e^{t^3}$ (use chain rule)

$$y' = 3t^2 e^{t^3}$$

(h) Find all local maxima and minima of $y = \frac{x^4}{2} - 2x^3 + x^2$.

$y' = 2x^3 - 6x^2 + 2x$

$y' = 2x(x^2 - 3x + 1)$

$y' = 0$ at $x = 0$ and $x^2 - 3x + 1$.

using roots of a quadratic we get

$x = 0$ and $x = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

$y'' = 6x^2 - 12x + 2$

Put $x = 0, \frac{3}{2} - \frac{\sqrt{5}}{2}, \frac{3}{2} + \frac{\sqrt{5}}{2}$ into y'' and you get:

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	y''	MAX OR MIN
$x = 0$	2	MIN
$x = \frac{3}{2} - \frac{\sqrt{5}}{2}$	-1.7	MAX
$x = \frac{3}{2} + \frac{\sqrt{5}}{2}$	11.71	MIN

Looks a bit like a typo.... the quadratic was probably meant to factorize more easily so it could be solved quickly.

2. Integration **(10 marks)**

Find the following integrals

(a) $\int \frac{1}{x^2} dx$ $\int x^{-2} dx = -x^{-1} + c$

(b) $\int 2e^{3x} dx$ $\frac{2}{3} e^{3x} + c$

(c) $\int \cos(3x) dx$ $\frac{1}{3} \sin(3x) + c$

(d) $\int \left(\frac{1}{x} - \sin(2x)\right) dx$ $\ln(x) + \frac{1}{2} \cos(2x) + c$

(e) $\int_0^1 \sqrt{2x} dx$ $= \sqrt{2} \int_0^1 x^{1/2} dx = \sqrt{2} \left[F(1) - F(0)\right]$

where $F(x) = \int x^{1/2} dx = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}$

Hence: $\sqrt{2} \left\{ \frac{2}{3} - 0 \right\} = 0.943$

(f) $\int_{-1}^{1}(2e^x - x^2)dx$ $= F(1) - F(-1)$

where $F(x) = \int 2e^x - x^2 dx = 2e^x - \frac{1}{3}x^3$

Hence: $\left(2e^1 - \frac{1}{3}\right) - \left(2e^{-1} + \frac{1}{3}\right) = 2e - 2/3 - 2/e$

$= 4.034$

(g) Using integration by substitution to show that

$$\int 2te^{t^2}dt = e^{t^2} + C.$$

$u = t^2$ $\quad \frac{du}{dt} = 2t \Rightarrow dt = \frac{du}{2t}$

$\int 2te^{t^2}dt = \int 2te^u \frac{du}{2t} = \int e^u du = e^u = e^{t^2}$

(h) Find the average of the function $f(t) = \frac{t^2}{2}$ across the interval $[-1, 1]$.

$\frac{1}{1-(-1)} \int_{-1}^{+1} t^2/2 \, dt = \frac{1}{2}\left[F(1) - F(-1)\right]$

where $F(t) = \int t^2/2 \, dt = t^3/6$

Hence: $\frac{1}{2}\left[\frac{1}{6} - \frac{(-1)^3}{6}\right] = \frac{1}{6}$

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3. Vectors **(5 marks)**

Given

$$a = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

Find

(a) $|b|$

$$\sqrt{(-1)^2+3^2+2^2} = \sqrt{14} = 3.742$$

(b) $a + 2b - 3c$

$$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 9 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

(c) $|a + c|$

$$\left|\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}\right| = \sqrt{4+16+4} = \sqrt{24} = 4.899$$

(d) $b \cdot c$

$$-1(0) + 3(3) + 2(1) = 0 + 9 + 2 = 11$$

(e) the angle θ between c and b

$$|c| = \sqrt{0^2+3^2+1^2} = \sqrt{10}$$

$$\cos\theta = \frac{\underset{\sim}{b} \cdot \underset{\sim}{c}}{|\underset{\sim}{b}||\underset{\sim}{c}|} = \frac{11}{\sqrt{14}\sqrt{10}}$$

$$\cos\theta = 0.9297 \implies \theta = 21.612^\circ$$

SPARE PAGE FOR EXTRA ANSWERS

Cross out rough working that you do not want marked.
Specify the question number for work that you do want marked.

Formula sheet

Function $f(x)$	Derivative $f'(x)$	Indefinite integral $\int f(x)\,dx$
k (constant)	0	$kx + c$
x	1	$\frac{1}{2}x^2 + c$
x^2	$2x$	$\frac{1}{3}x^3 + c$
x^n where $n \neq -1$	nx^{n-1}	$\frac{1}{n+1}x^{n+1} + c$
$\frac{1}{x}$ $(= x^{-1})$	$-x^{-2}$ $(= -\frac{1}{x^2})$	$\ln\lvert x\rvert + c$
$\sin x$	$\cos x$	$-\cos x + c$
$\sin(ax+b)$	$a\cos(ax+b)$	$-\frac{1}{a}\cos(ax+b) + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\cos(ax+b)$	$-a\sin(ax+b)$	$\frac{1}{a}\sin(ax+b) + c$
$\operatorname{cosec}(ax+b)$	$-a\operatorname{cosec}(ax+b)\cot(ax+b)$	$\frac{1}{a}\ln(\operatorname{cosec}(ax+b) - \cot(ax+b)) + c$
$\sec(ax+b)$	$a\sec(ax+b)\tan(ax+b)$	$\frac{1}{a}\ln(\sec(ax+b) + \tan(ax+b)) + c$
$\tan^{-1}(x)$	$\dfrac{1}{1+x^2}$	$x\tan^{-1}(x) - \frac{1}{2}\ln(x^2+1)$
e^x	e^x	$e^x + c$
e^{kx}	ke^{kx}	$\frac{1}{k}e^{kx} + c$
$\ln x$	$\frac{1}{x}$	$x\ln x - x + c$
$\ln(kx)$	$\frac{1}{x}$	$x\ln(kx) - x + c$

Integration by parts

$$\int u\left(\frac{dv}{dx}\right)dx = uv - \int\left(\frac{du}{dx}\right)v\,dx$$

Derivative Rules:

Product Rule: if $y = uv$, then $y' = u'v + uv'$

Quotient Rule: if $y = u/v$, then

$$y' = \frac{(vu' - uv')}{v^2}$$

Chain Rule: if $y = y(z)$ and $z = z(x)$, then

$$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$