School of Mathematics and Statistics
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ENGR 123 **Assignment 2**

1 *Propositions and quantifiers* Let $x, y \in \{2, 3, 4\}$. For the following propositions provide a proof or counterexample to the statements below:

- $P_0(x, y)$: $x < y$
- $P_1(x, y)$: both x and y are even
- $P_2(x, y)$: $x \times y > 7$
- $P_3(x, y)$: x is even and $x + y$ is a multiple of x.
- $P_4(x, y)$: $x \neq y$

(a) for all propositions P in $P_1, \ldots, P_5$,
$[\exists x \, \forall y \, P] \rightarrow [\forall y \, \exists x \, P]$ *(3 marks)*

Solution: TRUE by truth table below:

	P_0	P_1	P_2	P_3	P_4
$\forall y \, \exists x \, P_i$	0	0	1	0	1
$\exists x \, \forall y \, P_i$	0	0	1	0	0

The implication is TRUE, since for every column of the truth-table above, the entry in the second row implies the entry in the first (i.e. the statements $0 \rightarrow 0, 1 \rightarrow 1$ and $0 \rightarrow 1$ are all TRUE).

(b) for all propositions P in $P_1, \ldots, P_5$,
$[\forall x \, \exists y \, P] \rightarrow [\exists y \, \forall x \, P]$ *(3 marks)*

Solution: FALSE by counterexample:

	P_0	P_1	P_2	P_3	P_4
$\exists y \, \forall x \, P_i$	0	0	1	0	0
$\forall x \, \exists y \, P_i$	0	0	1	0	1

The second rows fails to imply the first in the fifth column, since $1 \rightarrow 0$ is a FALSE implication.

2 *Arguments*

(a) Let $A(x, y) =$"x agrees with y".
Rewrite the following statement and it negation formally. Rewrite the negation in English.

Everybody agrees with someone. *(3 marks)*

Solution: $\forall x \; \exists y \; A(x, y)$.
The negation is $\exists x \; \forall y \; \neg A(x, y)$, which in English is "someone agrees with no one" or perhaps "someone disagrees with everybody"

(b) Using A from the previous question, and letting $D(x, y) =$"x demurs to y", rewrite the following statement.
Somebody demurs to someone who everyone agrees with. *(3 marks)*

Solution: $(\exists x)(\exists y)[D(x, y) \wedge (\forall z) A(z, y)]$ or alternatively $(\exists x)(\exists y)(\forall z)(D(x, y) \wedge A(z, y))$.

(c) Rewrite the following statement.
Everyone demurs to exactly one person. *(3 marks)*

Solution: $(\forall x)(\exists y)[D(x, y) \wedge (D(x, z) \rightarrow z = y)]$

3 *Proofs*

Rewrite the following arguments formally and state which are valid/invalid.

Dictionary Use names for individuals (Mac and Ashley) and the following predicates
LovesFC(a) = "a loves fish and chips"; LikesTS(a) = "a likes tomato sauce";
Loud(a) = "a is a loud flier"; LikesCold(a) = "a likes the cold";
CITui(a) = "a is a Chatham island Tui"
Bird(a) = "a is a bird"; Kakapo(a) = "a is a Kakapo"

(a)
All lovers of fish and chips like tomato sauce.
Ashley loves fish and chips.

Ashley likes tomato sauce.

(3 marks)

Solution:

$(\forall x)[\text{LovesFAC}(x) \to \text{LikesTS}(x)]$
$\text{LovesFAC}(\mathit{Ashley})$ **Valid**

$\text{LikesTS}(\mathit{Ashley})$

(b)
No loud fliers like the cold.
All Chatham island Tui are loud fliers.

No Chatham island Tui like the cold.

(3 marks)

Solution:

$(\forall x)[\text{Loud}(x) \to \neg\text{LikesCold}(x)]$
$(\forall x)[\text{CITui}(x) \to \text{Loud}(x)]$ **Valid**

$\forall x)[\text{CITui}(x) \to \neg\text{LikesCold}(x)]$

(c)
All kakapo are birds.
Mac is a bird.

Mac is a kakapo.

(3 marks)

Solution:

$\forall x[\text{Kakapo}(x) \to \text{Bird}(x)]$
Bird(Mac) **Invalid**

Kakapo(Mac)

4 *Rearrangement*
Using this dictionary
Python(s) = Codes in Python; Indent(s) = Likes indenting code;
Baking(s) = Finds baking agreeable; Dishes(s) = Likes clean dishes;
Tramping(s) = Enjoys tramping; Outdoors(s) = Likes the outdoors.

rewrite the following argument to show that the conclusion follows logically.
That is, reorder the premises, and rewrite statements as "if-then's" or contrapositives where necessary. *(6 marks)*

1. If you don't enjoy tramping, then you will like dirty dishes.
2. Everyone that enjoys tramping likes the outdoors.
3. No one that likes indenting code finds baking disagreeable.
4. Everyone that codes in python likes indenting code.
5. Only those that find baking disagreeable like dirty dishes.

Everyone that codes in python likes the outdoors.

Solution:

Let s = "someone"

4.	$(\forall s)[\text{Python}(s) \rightarrow \text{Indent}(s)]$
3.	$(\forall s)[\text{Indent}(s) \rightarrow \text{Baking}(s)]$
5 cp.	$(\forall s)[\text{Baking}(s) \rightarrow \text{Dishes}(s)]$
1 cp.	$(\forall s)[\text{Dishes}(s) \rightarrow \text{Tramping}(s)]$
2.	$(\forall s)[\text{Tramping}(s) \rightarrow \text{Outdoors}(s)]$
	$(\forall s)[\text{Python}(s) \rightarrow \text{Outdoors}(s)]$