School of Mathematics and Statistics
Te Kura Mātai Tatauranga

ENGR 123 **Assignment 1**

1 *Proofs using truth-tables*

(a) Use a truth-table to show that
$P \vee Q \equiv \neg P \to Q$. *(2 marks)*

(b) Construct truth tables for the following propositions, and determine whether each is a tautology, contradiction, or contingent:

(i) $\neg P \to (P \vee Q)$ *(2 marks)*

(ii) $[(Q \wedge R) \to P] \to (\neg P \to \neg R)$ *(3 marks)*

2 *Translations*

(a) Using the propositional variable t to mean *she drinks tea* and c to mean *she drinks coffee* and h to mean *she drinks hot chocolate*, turn the following sentences into compound propositions:

(i) She drinks both coffee and hot chocolate, but not tea. *(1 mark)*

(ii) Drinking hot chocolate implies she drinks tea or coffee, but not both! *(2 marks)*

(b) Using the same variables, turn the following propositions directly into English:

(i) $\neg(t \wedge h)$ *(2 marks)*

(ii) $(t \vee c) \to h$ *(2 marks)*

3 *Building circuits with $NAND$-gates*

The truth-table for a $NAND$-gate is

P	Q	$\neg(P \wedge Q)$
0	0	1
0	1	1
1	0	1
1	1	0

Construct the following logic gates, using only $NAND$-gates:

(a) AND *(2 marks)*

(b) OR *(2 marks)*

4 *Minesweeper*

In the following 3×3 grid, there is at most 1 mine in each uncovered square, and the number in a square is the number of mines in neighbouring squares (maximum: 8).

Now, four squares have been revealed, and there is no mine in them. Let P_i be the proposition "square S_i has a mine".

S_1	1	0
S_2	4	2
S_3	S_4	S_5

(a) What does the "2" tell you (without considering the "1" and "4")? Write the proposition using notation from class. *(1 mark)*

(b) What does the "1" tell you (without considering the "2" and "4")? Write the proposition using notation from class. *(1 mark)*

(c) What does the "4" tell you (without considering the "1" and "2")? Write the proposition using notation from class. *(2 marks)*

(d) Write down a compound proposition that expresses the combined information in the "1", "2" and "4". *(1 mark)*

(e) Can you decide the truth-values of propositions $P_1, \ldots, P_5$ using the information provided? List all possible solutions. *(2 marks)*

5 *Add to multiply*

Construct a circuit that multiplies a double-digit binary number by three, using only half-adders, which were described in class. Please use a dotted line for the result digit, and a solid line for the carry digit. *(5 marks)*

		P	Q
×		1	1
T	U	V	W

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ENGR 123 **Tutorial 1**

1 *Proofs using truth tables*

(a) Show that $(P \to Q) \wedge (Q \to P)$ and $P \leftrightarrow Q$ are logically equivalent.

(b) Construct truth tables for the following propositions, and determine whether each is a tautology, contradiction, or contingent:

(i) $Q \leftrightarrow (\neg Q \wedge P)$
(ii) $\neg Q \to (Q \to P)$
(iii) $(P \to Q) \vee R$
(iv) $P \to (Q \to R)$

2 *Building circuits with NAND-gates*

Construct the following logic gates, using only $NAND$-gates, and check your answers using the "laws of logic". :

(a) NOT

(b) $IMPLIES$

3 *Knights and Knaves.*

Jeff is either a knight or a knave. Knights always tell the truth; knaves always lie.

Someone asks Jeff: "Are you a knight or a knave?"

He replies: "If I'm a knight, then I code in binary".

(a) Introduce the propositions

$$P = \text{"Jeff is a knight"}$$
$$Q = \text{"Jeff codes in binary"}$$

Translate Jeffs reply into propositional logic.

(b) Use a truth-table to show that Jeff must be a knight (and that he therefore codes in binary).

4 *Adders*

Construct a circuit that adds two double-digit binary numbers, using the half- and full-adder described in class.

$$\begin{array}{ccc} & \text{P} & \text{Q} \\ + & \text{R} & \text{S} \\ \hline \text{T} & \text{U} & \text{V} \end{array}$$