School of Mathematics and Statistics

Te Kura Mātai Tatauranga

ENGR 123 **Assignment 3**

1 For each relation on the given set, determine whether it is reflexive, symmetric, antisymmetric and/or transitive.

In each case where the relation doesn't have a property, give a short reason why or an example e.g. it isn't reflexive because ...

(a) On the set of people, $(a, b) \in R$ iff a and b share a parent in common. *(4 marks)*

(b) On the set of 2×2 matrices, $(a, b) \in R$ iff a and b have different top-left entries (so they differ in row one, column one). *(4 marks)*

2 Give a direct proof that shows if x is odd, then x^2 is odd. *(4 marks)*

3 For the equivalence relation on $\{0, 1, 2, 3, 4\}$ defined as

$$\{(x, y) : x - y \text{ is a multiple of } 3\}$$

(a) Write it out as a list of ordered pairs *(2 marks)*

(b) List the cells in the associated partition. *(2 marks)*

4 Let $A = \{1, 2, \ldots, 12\})$. Draw the Hasse diagram of the divisibility relation on A i.e. (x, y) iff x is a factor of y iff y is a multiple of x. *(2 marks)*

5 Use induction to show that $\frac{n!}{2^n} > 1$ for all $n \geq 4$. *(6 marks)*

6 Consider the following pseudocode:

Algorithm 1: `ReverserHelper(L, S)`

input: sequences $\mathtt{L} = [\mathtt{l}_1, \ldots, \mathtt{l}_\mathtt{m}]$ and $\mathtt{S} = [\mathtt{s}_1, \ldots, \mathtt{s}_\mathtt{n}]$

if (`Length(L)` $= 0$) **then**
  return `S`

$\mathtt{L}' = [\mathtt{l}_2, \ldots, \mathtt{l}_\mathtt{m}]$

$\mathtt{S}' = [\mathtt{l}_1, \mathtt{s}_1, \ldots, \mathtt{s}_\mathtt{n}]$

return `ReverserHelper`$(\mathtt{L}', \mathtt{S}')$

(a) Use induction on `m` to show that *(4 marks)*

$$\mathtt{ReverserHelper}([\mathtt{l}_1, \ldots, \mathtt{l}_\mathtt{m}], [\mathtt{s}_1, \ldots, \mathtt{s}_\mathtt{n}]) = [\mathtt{l}_\mathtt{m}, \mathtt{l}_{\mathtt{m}-1}, \ldots, \mathtt{l}_1, \mathtt{s}_1, \mathtt{s}_2, \ldots, \mathtt{s}_\mathtt{n}]$$

for any sequences $[\mathtt{l}_1, \ldots, \mathtt{l}_\mathtt{m}]$ and $[\mathtt{s}_1, \ldots, \mathtt{s}_\mathtt{n}]$

(b) Explain how to use the previous result to show that if $\mathtt{L} = [\mathtt{l}_1, \ldots, \mathtt{l}_\mathtt{m}]$ and

`Reverse(L)` $:=$ `ReverseHelper(L, [])`

then `Reverse(L)` returns $[\mathtt{l}_\mathtt{m}, \ldots, \mathtt{l}_1]$. *(2 marks)*

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ENGR 123 **Tutorial 3**

1 For each relation on the given set, determine whether it is reflexive, symmetric, antisymmetric and/or transitive.

In each case where the relation doesn't have a property, give a short reason why or an example e.g. it isn't reflexive because ...

(a) On the set of people, $(a, b) \in R$ iff b is a maternal ancestor of a.

(b) On the set of 2×2 matrices, a and b have the same determinant.

2 For the partition $\{0, 2\}, \{4\}, \{1, 3, 5\}$ of $\{0, 1, 2, 3, 4, 5\}$, list the ordered pairs in the equivalence relation produced by the partition.

3 Let $A = \mathcal{P}(\{j, k, l\})$ be the powerset (the set of all subsets) of $\{j, k, l\}$. Draw the Hasse diagram of the subset relation $\subset$ on A.

4 Use induction to show that

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n} = \frac{2^{n+1} - 1}{2^n}$$

5 Consider the following pseudocode:

Algorithm 1: `X(A)`

```
input: sequence A = (a_1, ..., a_n) of n distinct numbers, where n is a power of two
if (n = 1) then
    return a_1
else
    c = X(a_1, ..., a_{n/2})
    d = X(a_{n/2+1}, ..., a_n)
    if (c > d) then
        return c
    else
        return d
```

(a) What does algorithm `X(A)` do?

(b) Let $C(n)$ be the number of comparisons `X()` performs on a list of length n (where $n = 2^k$ is a power of two). Write down a formula for $C(n)$ and prove your answer using induction.

(c) Is it possible to design a more efficient algorithm that performs the same task (that is, one which uses fewer comparisons)? Why or why not?