

ENGR123 Test One
 45 minutes. 6 questions.
 30 marks total
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Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last two pages.

1. Complete the following truth table [6 marks]

P	Q	R	$P \implies \neg(Q \vee \neg R)$	$(P \wedge Q) \iff (R \implies Q)$
0	0	0	1 0 1 1	0 0 1
0	0	1	1 1 0 0	0 1 0
0	1	0	1 0 1 1	0 0 1
0	1	1	1 0 1 0	0 0 1
1	0	0	0 0 1 1	0 0 1
1	0	1	1 1 0 0	0 1 0
1	1	0	0 0 1 1	1 1 1
1	1	1	0 0 1 0	1 1 1

$\underline{\underline{4}}$ $\underline{\underline{3}}$ $\underline{\underline{2}}$ $\underline{\underline{1}}$ $\underline{\underline{1}}$ $\underline{\underline{3}}$ $\underline{\underline{2}}$

2. Consider the following (short) jumbled argument about birds, using the predicate P for parrot, G for gigantic and K for Kea:

- (1) All parrots are gigantic;
- (2) Nothing gigantic is fearful;
- (3) All kea are parrots.

(a) Rewrite each statement using predicates. [3 marks]

(b) State the contrapositive version of (2). [2 marks]

(c) Derive a conclusion, and order all the statements so that the conclusion follows logically from the premises. [2 marks]

a)

$$(1) \forall x (P(x) \rightarrow G(x))$$

$$(2) \forall y (G(y) \rightarrow \neg F(y)) \text{ OR } \forall y (F(y) \rightarrow \neg G(y))$$

$$(3) \forall z (K(z) \rightarrow P(z))$$

b) alternative in (2)

c)

(3)
(2)
(1)

$$\overline{\forall a (K(a) \rightarrow \neg F(a))}$$

No kea are fearful.

3. Determine the truth values of the following statements, provide a brief explanation in each case. [4 marks]

(a) $(\exists a \in \mathbb{N})(\forall b \in \mathbb{N})(a^2 > b)$

(b) $(\forall b \in \mathbb{N})(\exists a \in \mathbb{N})(a^2 > b)$

(a) False. Suppose $b = a^2 + 1$. Clearly $a^2 > a^2 + 1$ is false. So no a exists that works for all b .

(b) True. In general, let $a = b + 1$.
 $(b + 1)^2 > b$ is true when $b^2 + b + 1 > 0$
 which is true for all $b \in \mathbb{N}$.

4. What is the negation of

[2 marks]

$$(\exists a \in \mathbb{N})(\forall b \in \mathbb{N})(a^2 > b)$$

$$\begin{aligned} & \neg (\exists a \in \mathbb{N})(\forall b \in \mathbb{N})(a^2 > b) \\ \equiv & (\forall a \in \mathbb{N})(\exists b \in \mathbb{N})(a^2 \leq b) \\ \equiv & (\forall a \in \mathbb{N})(\exists b \in \mathbb{N}) \neg (a^2 > b) \\ \equiv & (\forall a \in \mathbb{N})(\exists b \in \mathbb{N})(a^2 \leq b) \end{aligned}$$

5. (a) What properties must a relation satisfy to be a partial order? [3 marks]
- (b) Let R be the relation on the set of movie actors, where $(a, b) \in R$ iff they have acted in a movie together.
- Show that R is reflexive and symmetric, but not transitive. [3 marks]
- (c) Explain why $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$ is not a partition of $\{1, 2, 3, 4, 5, 6\}$ [2 marks]

(a) Reflexive, Antisymmetric & Transitive.

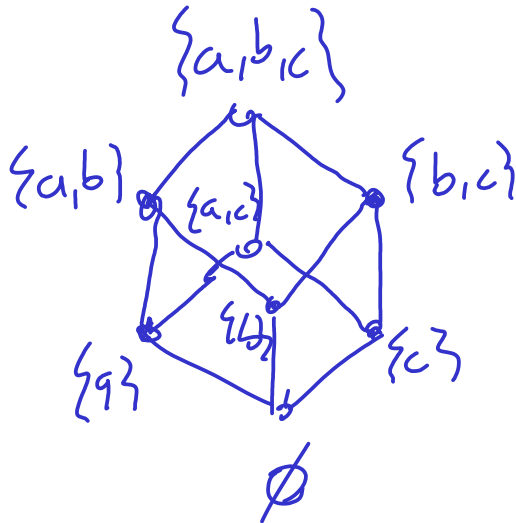
(b) Reflexive: movie actors have all acted in a movie with themselves.

Symmetric: If a & b have acted together, so have b & a !

Not transitive: Could be different movies.

6. Let $X = \mathcal{P}(\{a, b, c\})$ be the powerset of $\{a, b, c\}$.

Draw a Hasse diagram of the partial order R on X , where $(A, B) \in R$ iff $A \subseteq B$. [3 marks]



Rough working page

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
 $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
 $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \rightarrow Q \equiv \neg P \vee Q$
4. Commutative laws:
 $P \wedge Q \equiv Q \wedge P$
 $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
 $P \wedge P \equiv P$
 $P \vee P \equiv P$
6. Distributive laws:
 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
 $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
 $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
 $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$
9. Tautology: if \mathbb{T} is a tautology, then
 $P \vee \mathbb{T} \equiv \mathbb{T}$
 $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if \mathbb{F} is a contradiction, then
 $P \vee \mathbb{F} \equiv P$
 $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\frac{P \quad P \rightarrow Q}{Q}$$

- *Modus tollens.*

$$\frac{P \rightarrow Q \quad \neg Q}{\neg P}$$

- *Transitivity.*

$$\frac{P \rightarrow Q \quad Q \rightarrow R}{P \rightarrow R}$$

- *Contrapositive.*

$$\frac{P \rightarrow Q}{\neg Q \rightarrow \neg P}$$

Quantifiers

- *Universal* All P's are Q's

$$\forall x(P(x) \rightarrow Q(x))$$

- *Universal* Some P's are Q's

$$\exists x(P(x) \wedge Q(x))$$

- *Negating quantifiers*

$$\neg \forall x[R(x)] \equiv \exists x[\neg R(x)]$$

$$\neg \exists x[R(x)] \equiv \forall x[\neg R(x)]$$