

ENGR123 Test One
45 minutes. 6 questions.
30 marks total
15th August 2019

Surname:
First names:
ID Number:

Please use the spaces provided in this test booklet to give your answers. Attempt all questions. Blank pages for rough work are provided toward the end. A formula sheet is on the last two pages.

1. Complete the following truth table [6 marks]

P	Q	R	$P \implies \neg(Q \vee \neg R)$	$(P \wedge Q) \iff (R \implies Q)$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

2. Consider the following (short) jumbled argument about birds, using the predicate P for parrot, G for gigantic and K for Kea:

- (1) All parrots are gigantic;
- (2) Nothing gigantic is fearful;
- (3) All kea are parrots.

- (a) Rewrite each statement using predicates. **[3 marks]**
- (b) State the contrapositive version of (2). **[2 marks]**
- (c) Derive a conclusion, and order all the statements so that the conclusion follows logically from the premises. **[2 marks]**

3. Determine the truth values of the following statements, provide a brief explanation in each case. [4 marks]

(a) $(\exists a \in \mathbb{N}) (\forall b \in \mathbb{N}) (a^2 > b)$

(b) $(\forall b \in \mathbb{N}) (\exists a \in \mathbb{N}) (a^2 > b)$

4. What is the negation of

[2 marks]

$$(\exists a \in \mathbb{N}) (\forall b \in \mathbb{N}) (a^2 > b)$$

5. (a) What properties must a relation satisfy to be a partial order? **[3 marks]**
- (b) Let R be the relation on the set of movie actors, where $(a, b) \in R$ iff they have acted in a movie together.
- Show that R is reflexive and symmetric, but not transitive. **[3 marks]**
- (c) Explain why $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$ is not a partition of $\{1, 2, 3, 4, 5, 6\}$ **[2 marks]**

6. Let $X = \mathcal{P}(\{a, b, c\})$ be the powerset of $\{a, b, c\}$.

Draw a Hasse diagram of the partial order R on X , where $(A, B) \in R$ iff $A \subseteq B$. **[3 marks]**

Rough working page

List of laws of logic

1. Double negation: $P \equiv \neg\neg P$
2. De Morgan's laws:
 $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
 $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
3. $P \rightarrow Q \equiv \neg P \vee Q$
4. Commutative laws:
 $P \wedge Q \equiv Q \wedge P$
 $P \vee Q \equiv Q \vee P$
5. Idempotent laws:
 $P \wedge P \equiv P$
 $P \vee P \equiv P$
6. Distributive laws:
 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
 $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7. Associative laws:
 $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
 $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
8. Contrapositive: $(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$
9. Tautology: if \mathbb{T} is a tautology, then
 $P \vee \mathbb{T} \equiv \mathbb{T}$
 $P \wedge \mathbb{T} \equiv P$
10. Contradiction: if \mathbb{F} is a contradiction, then
 $P \vee \mathbb{F} \equiv P$
 $P \wedge \mathbb{F} \equiv \mathbb{F}$

Some rules of inference

- *Modus ponens.*

$$\frac{P \quad P \rightarrow Q}{Q}$$

- *Modus tollens.*

$$\frac{P \rightarrow Q \quad \neg Q}{\neg P}$$

- *Transitivity.*

$$\frac{P \rightarrow Q \quad Q \rightarrow R}{P \rightarrow R}$$

- *Contrapositive.*

$$\frac{P \rightarrow Q}{\neg Q \rightarrow \neg P}$$

Quantifiers

- *Universal* All P's are Q's

$$\forall x(P(x) \rightarrow Q(x))$$

- *Universal* Some P's are Q's

$$\exists x(P(x) \wedge Q(x))$$

- *Negating quantifiers*

$$\neg \forall x[R(x)] \equiv \exists x[\neg R(x)]$$

$$\neg \exists x[R(x)] \equiv \forall x[\neg R(x)]$$