ENGR123 Logic Test
90 minutes.
7 questions.
45 marks total
15th August 2019

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First names:	Steven
ID Number:	On the low side

Please use the spaces provided in this test booklet to give your answers.
Attempt all questions.
Blank pages for rough work are provided toward the end.
The test is open-book, but not open-internet. Only the course homepage or blackboard is acceptible online material to access during the test.

The test consists of 7 questions and is out of 45.

Question	Topic	Out of	Mark
1	Logic	7	
2	Logic	5	
3	Relations	10	
4	Proofs	6	
5	Recursion	7	
6	Eulerian circuits	5	
7	Insight	5	

1. Consider the following list of statements

(1)	Nothing beautiful lacks in wonder.	$\forall x\,[\text{Beaut}(x) \to \text{Wonder}(x)]$
(2)	Anything that makes us unhappy lacks in wonder.	$\forall x\,[\text{Wonder}(x) \to \text{Happy}(x)]$
(3)	All islands in NZ are beautiful.	$\forall x\,[\text{Isle}(x) \to \text{Beaut}(x)]$
(4)	Nothing that makes us happy is to be sniffed at.	$\forall x\,[\text{Happy}(x) \to \text{Sniff}(x)]$
(5)	All beautiful things are appreciated.	$\forall x\,[\text{Beaut}(x) \to \text{Appreciate}(x)]$
(6)	Everything wondrous is beautiful.	$\forall x\,[\text{Wonder}(x) \to \text{Beaut}(x)]$

(a) Rewrite each statement using quantifiers, and these predicates **[4 marks]**

- Appreciate(x):x is appreciated
- Beaut(x):x is beautiful
- Happy(x):x makes us happy
- Isle(x):x is an island in NZ
- Sniff(x):x is to be sniffed at
- Wonder(x):x is wondrous

You can use the boxes above if you wish

(b) Four of the statements from the list are the premisses of an argument about islands (this is a hint about where to begin).

Put those four in the correct order (the numbers will suffice), and state the conclusion (which isn't in the list) in English. **[3 marks]**

(3), (1), (2), (4) All islands in NZ are not to be sniffed at.

OR

(3), (1), (6), (5) All islands in NZ are appreciated.

2. The following four statements are all true. Provide a brief explanation why in each case.

[5 marks]

(a) $(\forall y \in \mathbb{Z})\ (\exists x \in \mathbb{Z})\ (xy \geq y)$

(b) $(\exists x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ (xy \geq y)$

(c) $(\exists a \in \mathbb{Z})\ (\forall b \in \mathbb{Z})\ (a \text{ is a multiple of } b)$

(d) $(\forall b \in \mathbb{Z})\ (\exists a \in \mathbb{Z})\ (x^2 + bx + a = 0 \text{ has 2 distince integer roots/solutions})$

3. Consider the following ten parks in the Wellington region.

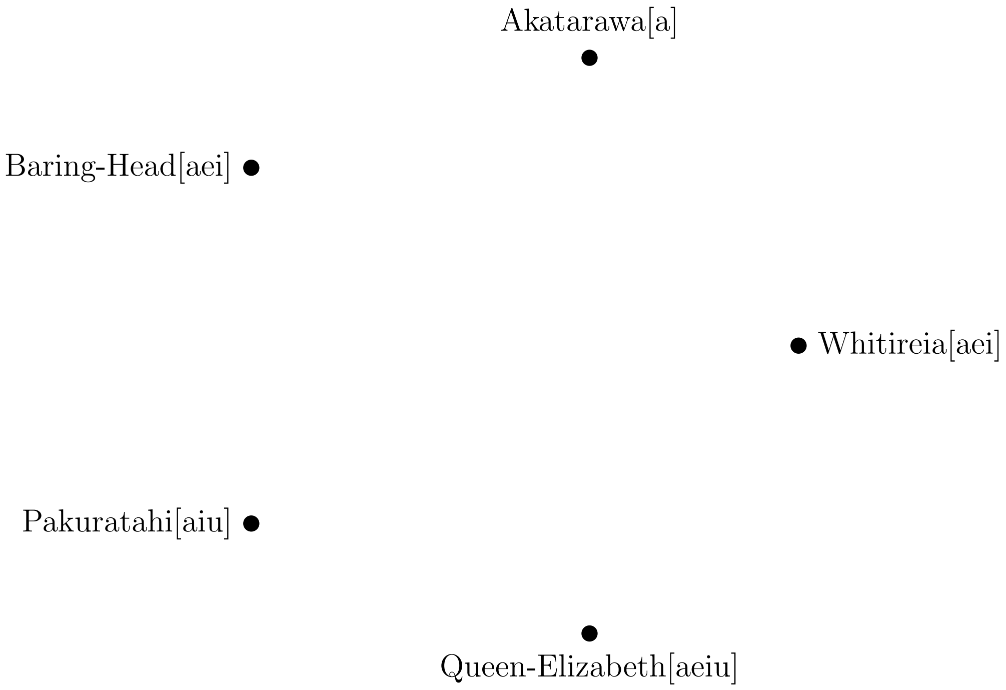
- Akatarawa
- Baring-Head
- Battle-Hill
- Belmont
- East-Harbour
- Kaitoke
- Pakuratahi
- Queen-Elizabeth
- Te Awarua-o-Porirua
- Whitireia

For this question, two parks are related by R if, ignoring repeated letters, their names contain at least three distinct vowels in common e.g. Kaitoke and Whitireia are related as both contain the vowels 'a', 'e' and 'i'.

(a) Using the diagram below, draw a directed graph to show how five of the parks (Akatarawa, Baring-Head, Pakuratahi, Queen-Elizabeth and Whitireia) are related by R.

The letters in square brackets are the unique vowels from the name in alphabetical order, this should help!

[2 marks]



(b) For each property below, state whether or not the relation R *on the ten parks* has the relevant property.

Where it does not, give an example why e.g. it is not reflexive because ...

[4 marks]

Reflexive:

Symmetric:

Anti-Symmetric:

Transitive:

(c) Define another relation S on the ten parks as park x and y are related if they share the same first letter or last letter e.g. Baring-Head and Battle-Hill share the same first letter (B) so are related by S. Similarly Akatarawa and Whitireia share the same last letter (a) so are also related by S. **[4 marks]**

i. Surprisingly, S is an equivalence relation. List the cells of the associated partition.

ii. If we extended S to be a relation *on all place names* then S wouldn't be an equivalence relation.

- Which property wouldn't hold in this extension?

- Give an example, where the property you just mentioned doesn't hold.

4. Proof techniques.

   (a) Using a contrapositive proof, explain why if xy is even, then x or y is even. **[3 marks]**

   (b) Using a proof by contradiction, explain why if n^2 is *not* a multiple of 4, then n is odd.

   [3 marks]

5. Consider the following pseudocode:

```
Algorithm 1: Divit(A)
input: sequence A = (a_1, ..., a_n) of $n$ distinct non-zero numbers, where $n$ is a power of two
if (n = 2) then
    return a_1/a_2
else
    c = Divit(a_1, ..., a_{n/2})
    d = Divit(a_{n/2+1}, ..., a_n)
    return $c \times d$
```

(a) Let $C(n)$ be the number of arithmetic operations (addition, subtraction, multiplication or division) that the algorithm `Divit()` performs on a list of length n (where $n = 2^m$ is a power of two).

Use induction to prove that $C(n) = n - 1$

[4 marks]

Base Case:

Induction Hypothesis:

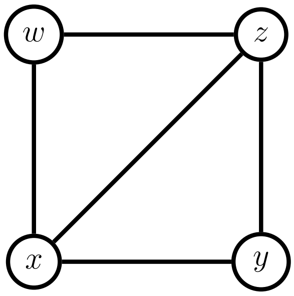
Induction Step:

(b) Give a (non-recursive) formula for the output of the algorithm, and using induction, prove that the output of `Divit(`$a_1, \ldots, a_n$`)` matches your formula.

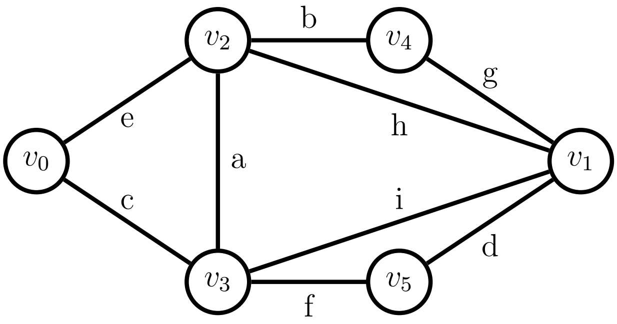
[3 marks]

6. (a) Fleury's algorithm finds an Eulerian circuit on certain graphs. Explain why this graph doesn't have an Eulerian circuit.

[2 marks]



(b) I've run Fleury's algorithm on the following graph



and the resulting Eulerian circuit is

$$v_1 - v_5 - v_3 - v_2 - v_0 - v_3 - v_1 - v_4 - v_2 - v_1$$

As explained in class, indicate where each bridge appears as the algorithm runs, whether taken or not.

$$v_1 \text{ ——— } v_5 \underset{\textit{Bridge}}{\text{ ——— }} v_3 \text{ ——— } v_2 \text{ ——— } v_0 \text{ ——— } v_3 \text{ ——— } v_1 \text{ ——— } v_4 \text{ ——— } v_2 \text{ ——— } v_1$$

[3 marks]

7. Suppose R and S are equivalence relations on A.

[5 marks]

(a) The set $R \cap S$ is a relation on A. Using definitions, prove that $R \cap S$ is an equivalence relation on A.

(b) Now explain why $R \cap S$ is an equivalence relation using partitions rather than definitions. Make sure to include a diagram to assist with your explanation.

Rough working page