ENGR123 Logic Test
90 minutes.
7 questions.
45 marks total
15th August 2019

Surname:	
First names:	
ID Number:	

Please use the spaces provided in this test booklet to give your answers.
Attempt all questions.
Blank pages for rough work are provided toward the end.
The test is open-book, but not open-internet. Only the course homepage or blackboard is acceptible online material to access during the test.

The test consists of 7 questions and is out of 45.

Question	**Topic**	**Out of**	**Mark**
1	Logic	7	
2	Logic	5	
3	Relations	10	
4	Proofs	6	
5	Recursion	7	
6	Eulerian circuits	5	
7	Insight	5	

1. Consider the following list of statements

(1)	Anything that makes us unhappy is not welcome	
(2)	Nothing we'd write a blog about is not welcome	
(3)	Nothing that makes us happy is to be sonneted	
(4)	Everything that is interesting we'd write a blog about	
(5)	Anything that is welcome we'd write a blog about.	
(6)	All things we'd write a blog about are apropriate.	

(a) Rewrite each statement using quantifiers, and these predicates **[4 marks]**

- Appropriate(x):x is appropriate
- Blog(x):we'd write a blog about x
- Happy(x):x makes us happy
- Interest(x):x is interesting
- Sniff(x):x is to be sniffed at
- Welcome(x):x makes us welcome

You can use the boxes above if you wish

(b) Four of the statements from the list are the premisses of an argument about interesting things (this is a hint about where to begin).

Put those four in the correct order (the numbers will suffice), and state the conclusion (which isn't in the list) in English. **[3 marks]**

2. The following four statements are all true. Provide a brief explanation why in each case.

[5 marks]

(a) $(\forall y \in \mathbb{Z})\ (\exists x \in \mathbb{Z})\ (xy \leq y)$

(b) $(\exists x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ (xy \leq y)$

(c) $(\exists a \in \mathbb{Z})\ (\forall b \in \mathbb{Z})$ (a is a multiple of b)

(d) $(\forall b \in \mathbb{Z})\ (\exists a \in \mathbb{Z})$ ($x^2 + ax + b = 0$ has 2 distinct integer roots/solutions)

3. Consider the following ten suburbs in the Wellington region.

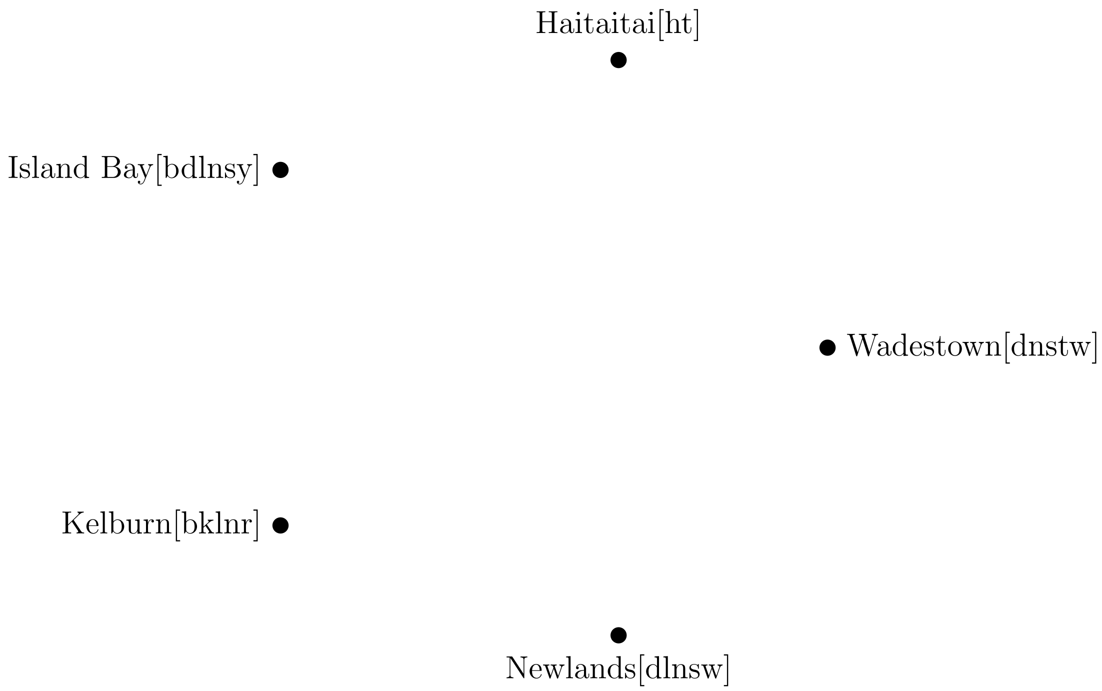
- Haitaitai
- Happy Valley
- Island Bay
- Karori
- Kelburn
- Khadallah
- Newlands
- Petone
- Titahi Bay
- Wadestown
- Wainuiomata

For this question, two suburbs are related by R if, ignoring repeated letters, their names contain at least three distinct consonants in common e.g.Newlands and Wadestown are related as both contain the consonants 'd', 'n' and 's'.

(a) Using the diagram below, draw a directed graph to show how five of the suburbs (Haitaitai, Island Bay, Kelburn, Newlands and Wadestown) are related by R.

The letters in square brackets are the unique consonants from the name in alphabetical order, this should help!

[2 marks]



(b) For each property below, state whether or not the relation R on the 10 suburbs has the relevant property.

Where it does not, give an example why e.g. it is not reflexive because ...

[4 marks]

Reflexive:

Symmetric:

Anti-Symmetric:

Transitive:

(c) Define another relation S on the ten suburbs as suburb x is related to suburb y if the first letter of x comes before y alphabetically e.g. the first letter of Happy Valley is H, which comes before I, the first lette of Island Bay. So Happy Valley is related to Island Bay. **[4 marks]**

i. The relation S isn't a partial order, which property doesn't hold.

ii. If we treated suburbs with the same first letter as equal, it would be a partial order.

- Which of the 10 suburbs would be treated the same as each other.

- Draw a Hasse diagram of the resulting partial order

4. Proof techniques.

(a) Using a contrapositive proof, explain why if $x^2 + y^2$ is odd, then x or y is odd. **[3 marks]**

(b) Using a proof by contradiction, explain why if $3n$ is *not* a multiple of 6, then n is an odd number. **[3 marks]**

5. Consider the following pseudocode:

Algorithm 1: `Minusit(A)`

input: sequence $\mathtt{A} = (\mathtt{a}_1, \ldots, \mathtt{a}_\mathtt{n})$ of n distinct non-zero numbers, where n is a power of two

if $(\mathtt{n} = 2)$ **then**
    return $\mathtt{a}_1 - \mathtt{a}_2$
else
    $\mathtt{c} = \mathtt{Minusit}(\mathtt{a}_1, \ldots, \mathtt{a}_{\frac{n}{2}})$
    $\mathtt{d} = \mathtt{Minusit}(\mathtt{a}_{\frac{n}{2}+1}, \mathtt{a}_\mathtt{n})$
    return $c + d$

(a) Let $C(n)$ be the number of arithmetic operations (addition, subtraction, multiplication or division) that the algorithm `Minusit()` performs on a list of length n (where $n = 2^m$ is a power of two).

Use induction to prove that $C(n) = n - 1$

[4 marks]

Base Case:

Induction Hypothesis:

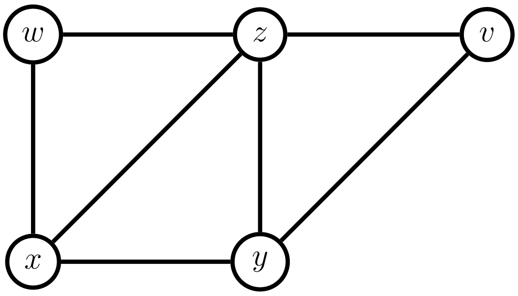
Induction Step:

(b) Give a (non-recursive) formula for the output of the algorithm, and using induction, prove that the output of $\texttt{Minusit}(\texttt{a}_1, \ldots, \texttt{a}_\texttt{n})$ matches your formula.

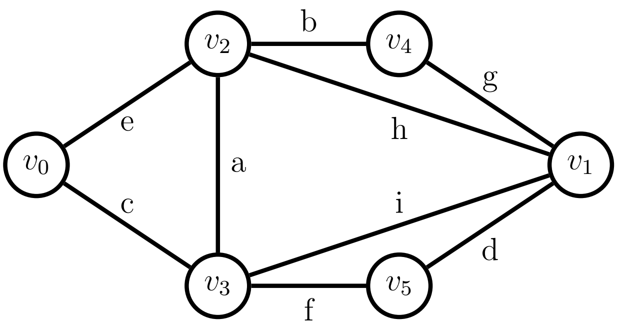
[3 marks]

6. (a) Fleury's algorithm finds an Eulerian circuit on certain graphs. Explain why this graph doesn't have an Eulerian circuit.

[2 marks]



(b) I've run Fleury's algorithm on the following graph



and the resulting Eulerian circuit is

$$v_0 - v_2 - v_3 - v_5 - v_1 - v_4 - v_2 - v_1 - v_3 - v_0$$

As explained in class, indicate where each bridge appears as the algorithm runs, whether taken or not.

v_0 ——— v_2 ——— v_3 ——— v_5 ——— v_1 ——— v_4 ——— v_2 ——— v_1 ——— v_3 ——— v_0

Bridge c

[3 marks]

7. Suppose R is a partial order on A.

[5 marks]

(a) Using definitions, prove that RR is a partial order on A.

(b) Now explain why RR is a partial order using Hasse diagrams rather than definitions. Make sure to include a diagram to assist with your explanation.

Rough working page